**Applications of the compound interest formula**

Quite often, three of the variables used in the compound interest formula are known and the fourth needs to be found.

**Finding P**

**worked example 6**

Aunt Freda leaves Thelma a legacy—some deposit stock that was invested for ten years at 11% p.a. compounded quarterly. The value of the cheque received was $38 478.36. Calculate the initial deposit.

**Steps**

1. Write the formula.
2. Identify \( P \), unknown.
3. Calculate \( i \) and \( n \).
4. Identify \( A \).
5. Substitute. \( 38 478.36 = P(1 + 0.0275)^{40} \)
6. Transpose to find \( P \).
7. Evaluate.

**Solution**

\[ A = P(1 + i)^n \]

\[ P = ? \]

\[ i = \frac{0.11}{4} = 0.0275 \]

\[ n = 10 \times 4 = 40 \]

\[ A = 38 478.36 \]

\[ 38 478.36 = P(1 + 0.0275)^{40} \]

\[ P = \frac{38 478.36}{2.959 873 987} \]

\[ P = \approx 13 000 \]

**Finding i (interest rate per period)**

**worked example 7**

When she first began it, Claudia’s business had takings of $228 000 p.a. Her takings twelve years later are $520 000. At what rate p.a. (correct to one decimal place) is her business growing?

**Steps**

1. Write the formula.
2. Identify \( P \).
3. Identify \( i \), unknown.
4. Identify \( n \).
5. Identify \( A \).
6. Substitute. \( 520 000 = 228 000(1 + i)^{12} \)
7. Divide by the principal (in this case, 228 000).

**Solution**

\[ A = P(1 + i)^n \]

\[ P = 228 000 \]

\[ i = ? \]

\[ n = 12 \]

\[ A = 520 000 \]

\[ 520 000 = 228 000(1 + i)^{12} \]

\[ \frac{520 000}{228 000} = (1 + i)^{12} \]

\[ (1 + i)^{12} = \approx 2.299 \]

\[ i = \frac{\log(2.299)}{12} \]

\[ i \approx 0.023 \]

\[ i \approx 2.3\% \]
If the compounding period is other than a year, we need to find $R$, the interest rate p.a. To do this we calculate $i \times \text{number of periods per year}$. So if we found $i = 0.03$ and there were four periods per year (that is, the interest was compounded quarterly), the annual interest rate would be $R = 0.03 \times 4 = 0.12$ or 12%.

**Finding $n$**

The algebra is complex, but questions where $n$ is unknown may be solved using trial and error or by the use of a spreadsheet.

### worked example 8

After how many years will a $450 stamp be worth at least $900 if it increases in value by 7.5% p.a.?

<table>
<thead>
<tr>
<th>Steps</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write the formula.</td>
<td>$A = P(1 + i)^n$</td>
</tr>
<tr>
<td>2. Identify $P$.</td>
<td>$P = 450$</td>
</tr>
<tr>
<td>3. Identify $i$.</td>
<td>$i = 0.075$</td>
</tr>
<tr>
<td>4. Identify $n$, unknown.</td>
<td>$n = ?$</td>
</tr>
<tr>
<td>5. Identify $A$.</td>
<td>$A = 900$</td>
</tr>
<tr>
<td>6. Substitute.</td>
<td>$900 = 450(1 + 0.075)^n$</td>
</tr>
<tr>
<td>7. Simplify the right side (in this case, evaluate $1 + 0.075$).</td>
<td>$900 = 450(1.075)^n$</td>
</tr>
<tr>
<td>8. Divide both sides by the principal (in this case, 450).</td>
<td>$\frac{900}{450} = (1.075)^n$</td>
</tr>
</tbody>
</table>

Rate $\approx 7.1\%$
1. How much will a consumer have to repay on a debt of $9000 after two years, if the 13.5% interest p.a. is compounded annually?

2. Calculate the total amount owing on a loan of $6250 after three years, if the 14% interest p.a. is compounded annually.

3. How much will a coin valued at $1500 be worth after two years if it appreciates in value by 10.5% p.a.?

4. Choose the correct answer.
   $4245 at 6.5% p.a. compound interest, compounded annually over eight years, will amount to:
   - A $4245(1 + 0.065)^8$
   - B $4245(1 + 0.65)^8$
   - C $4245(1 + 0.065) \times 8$
   - D $4245(1 + 0.065)^8 - 4245$
   - E $4245(1 + 0.065)

5. Choose the correct answer.
   If a loan of $5800 is made at 16% p.a. compounded every half-year, over six years the debt will grow to:
   - A $5800(1 + 0.16)^6$
   - B $5800(1 + 0.08)^6$
   - C $5800(1 + 0.08)^{12}$
   - D $5800(1 + 0.16)^{12}$
   - E $5800(1 + 0.8)^{12}$

6. Choose the correct answer.
   If the loan in Question 5 were compounded quarterly, the interest accrued would be:
   - A $5800(1 + 0.16)^6 - 5800$
   - B $5800(1 + 0.04)^{24}$
   - C $5800(1 + 0.04)^6$
   - D $5800(1 + 0.04)^{24} - 5800$
   - E $5800(1 + 0.4)^{24} + 5800$

7. Calculate the total amount owing after two years on a loan of $16 250 if the 11.25% interest p.a. is compounded:
   - (a) annually
   - (b) half-yearly.

---

9. Evaluate the left side.

10. Use a calculator or spreadsheet to try different values for $n$ until you find a satisfactory answer (in this case, until $1.075^n \geq 2$).

11. State the answer.
    After ten years the stamp will be worth at least $900.

---

**Exercise 1.4 Compound interest by formula**

1. How much will a consumer have to repay on a debt of $9000 after two years, if the 13.5% interest p.a. is compounded annually?

2. Calculate the total amount owing on a loan of $6250 after three years, if the 14% interest p.a. is compounded annually.

3. How much will a coin valued at $1500 be worth after two years if it appreciates in value by 10.5% p.a.?

4. Choose the correct answer.
   $4245 at 6.5% p.a. compound interest, compounded annually over eight years, will amount to:
   - A $4245(1 + 0.065)^8$
   - B $4245(1 + 0.65)^8$
   - C $4245(1 + 0.065) \times 8$
   - D $4245(1 + 0.065)^8 - 4245$
   - E $4245(1 + 0.065)$

5. Choose the correct answer.
   If a loan of $5800 is made at 16% p.a. compounded every half-year, over six years the debt will grow to:
   - A $5800(1 + 0.16)^6$
   - B $5800(1 + 0.08)^6$
   - C $5800(1 + 0.08)^{12}$
   - D $5800(1 + 0.16)^{12}$
   - E $5800(1 + 0.8)^{12}$

6. Choose the correct answer.
   If the loan in Question 5 were compounded quarterly, the interest accrued would be:
   - A $5800(1 + 0.16)^6 - 5800$
   - B $5800(1 + 0.04)^{24}$
   - C $5800(1 + 0.04)^6$
   - D $5800(1 + 0.04)^{24} - 5800$
   - E $5800(1 + 0.4)^{24} + 5800$

7. Calculate the total amount owing after two years on a loan of $16 250 if the 11.25% interest p.a. is compounded:
   - (a) annually
   - (b) half-yearly.
8 Tula borrows $5000 to buy a motorbike. How much will she have to repay to pay off the debt after one and a half years, if the 12% interest p.a. is compounded:
   (a) half-yearly?
   (b) quarterly?

9 How much interest is added over ten years to an account paying 9% interest p.a. on an initial sum of $45 800 if the interest is compounded:
   (a) half-yearly?      (b) quarterly?
   (c) monthly?

10 Nicola’s salary has increased by 7% p.a. over the past four years to $41 500 p.a. Calculate her salary four years ago, to the nearest dollar.

11 The number of people in an Australian town increases by 1.1% p.a. Its current population is 68 000. What was the population six years ago?

12 Calculate how much would need to be invested at 8% p.a. compounded each half-year to accumulate to $9600 in six years.

13 How much would Li have to deposit in order to receive $10 000 in seven years if she places her money in an account that pays 8.8% interest p.a., compounded quarterly?

14 Make up three different investment situations in which the money grows to about $100 000 over the time allocated, with monthly compounding. State the principal, the time, and the rate as a percent per annum.

15 Choose the correct answer.
   Costs in a business are growing at 8% p.a. Currently they run at $780 per week. Seven years ago they were:
   
   A $\frac{780}{1.08^7}$  
   B $780 \times 1.08^7$  
   C $780 \times 1.08^7 - 780$

16 Choose the correct answer.
   A deposit accumulates to $4500 in nine months at 12% p.a. compounded quarterly. The initial deposit was:
   
   A $\frac{4500}{1.12^3}$  
   B $4500 \times 1.04^3$  
   C $\frac{4500}{1.03^3}$
17 Choose the correct answer. $2800 accumulates to $4500 in three years. The percentage rate of interest p.a., if it is compounded annually, is:

A \( \frac{\sqrt[3]{4500}}{2800} \times 100\% \)  
B \( \frac{4500}{2800} \times 100\% \)  
C \( \frac{\sqrt[4]{4500}}{2800} \times 100\% \)  
D \( \left( \frac{\sqrt[3]{4500}}{2800} - 1 \right) \times 100\% \)  
E \( \left( \frac{\sqrt[4]{4500}}{2800} - 1 \right) \times 100\% \)

18 How much more will an investor get on an investment of $32 000 over four years in an account offering 8.8% p.a. if the interest is compounded weekly rather than annually? (Assume 52 weeks in a year.)

19 Calculate the interest in Question 18 if it is compounded daily.

20 How much would Sylvester have to deposit to receive $8000 in five years’ time if he places money in an account that pays 7.8% interest p.a., compounded half-yearly?

21 Sales of $26 900 grow to $78 000 in 11 years. Calculate the percentage growth p.a.

22 Costs of $6200 increase to $8000 in eight years. Calculate the percentage increase p.a.

23 Calculate the rate of interest p.a. that would allow $7900 to accumulate to $9000 in five years if interest is compounded each half-year.

24 After how many years will a $2400 sapphire ring be worth at least $8000 if it increases in value by 10.5% p.a.?

25 After how many years will a $40 000 block of land be worth at least $90 000 if it increases in value by 8.5% p.a.?

26 After how many years will a $3400 porcelain dinner set be worth at least $5000 if it increases in value by 9.7% p.a.?
People with large amounts of money invested in the stock market are likely to want to rush out and sell if the price of a particular stock is predicted to fall significantly, yet few home owners react in such a way as the values of homes fluctuate over time.

If calculations were posted daily on property values, it would be quite possible that a home could lose $10,000 of its value overnight. Over time, however, it’s most likely that growth in value would occur. Assuming a growth of just 5% a year, the property would rise in value by about 60% in 10 years, double in 13 years and triple in 18 years.

Home loan interest rates are the other side to buying a dream home. Unlike property values, there is generally no long-term trend up or down. A rise at the wrong time can make home owning very expensive. Purchasing property is not a good short-term investment. It is sometimes better to rent for a short period of time and to invest the deposit, and the difference between rent and what would be loan repayments, elsewhere.

Questions

1. Alex and Trinh want to purchase a $270,000 property. Most financial institutions will lend them only 90% of this amount. What deposit do Alex and Trinh need?

2. Alex and Trinh take out a $243,000 loan for a 25-year term at fixed interest of 7% p.a. Their monthly repayments are $1717, which remain the same for the 25-year period.
   (a) How much will they have paid for the house at the end of the 25 years, given that they paid stamp duty of $8000? Don’t forget to include the initial deposit.
   (b) How much interest will they have paid in total?
   (c) Find the average amount of interest in the monthly repayment of $1717.
3. It is generally considered that houses appreciate in value at a rate of approximately 5% per year. In some locations this rate of appreciation is greater. At the beginning of the first year the value of Alex and Trinh’s house is $270 000. Copy the following table and use compound interest to complete columns 2, 3 and 4.

<table>
<thead>
<tr>
<th>Year</th>
<th>Value at beginning of year (($)</th>
<th>Increase in value (($)</th>
<th>Value at end of year (($)</th>
<th>Maintenance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>270 000</td>
<td>270 000 × 0.05 = 13 500</td>
<td>283 500</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>283 500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Another cost of owning a house is maintenance. The approximate amount spent on maintenance per year is 1% of the house’s value that year. Complete column 5 of the table above by finding 1% of each of the end-of-year values.

5. After 4 years, Alex and Trinh decide to sell their house. The total cost to them so far is the initial value of the house, plus stamp duty, plus interest already paid, plus maintenance costs. Show that the total cost of the house is $333 756.

6. If Alex and Trinh only manage to sell the house for $320 000, slightly under its predicted value, what will be their profit/loss?

7. Four years ago, friends of Alex and Trinh, Kate and Mustafa, were also looking to buy a house valued at $270 000. Instead they decided to continue renting at $1200 per month. They invested the deposit of $27 000, then continued to invest the difference between their potential mortgage repayments ($1717 per month) and the rent.

(a) How much rent have they paid in total during the 4 years if rent has increased by 5% per year, so that the rent was $1200 per month in the first year, $1260 per month in the second, $1323 per month in the third and $1389 per month in the fourth?

(b) Their $27 000 investment, plus deposits, has mounted to $64 430 at the end of the 4 years. Subtract rent from this amount to find their profit/loss.

(c) Which couple is better off financially after the 4-year period, and by how much?

8. If Alex and Trinh sold their property after 15 years instead of 4 years, they still would make a loss on the property.

(a) Find the amount of the loss if the cost of the house is its initial value plus $8000 stamp duty plus interest of $214 060 and maintenance of $84 195, and if the house was sold for $561 310.

(b) If Alex and Trinh had taken the option that Kate and Mustafa took and rented for 15 years while investing the rest, they would lose, in total, $207 150. Financially, by what amount would they be better off by buying and selling rather than renting and investing?

Research

Compare three banks or home-lending institutions for a $120 000 home loan that you will be aiming to pay back in 10 years. Which institution offers the best deal, in your opinion? Write a report that includes the calculations and spreadsheets you use to reach your conclusion.
As soon as you take delivery of your brand new car it loses value. This loss of value is called depreciation and it affects products of many different types. Capital equipment used in manufacturing processes is one major example of a business application of depreciation.

Here we will consider two different types of depreciation: straight-line depreciation and reducing-balance depreciation. A business owner can choose whichever depreciation method will maximise benefit to the business.

**Straight-line depreciation**

For straight-line depreciation the equipment loses the same value each year. For example, a piece of equipment with an initial value of $100 000 might lose $10 000 in value each year. Over the course of 10 years it would lose all of its value. Straight-line depreciation amounts can be expressed as a dollar amount, as we have just seen, or as a percentage of the original value of the equipment. In this case that would be 10% of the original value.

**Reducing-balance depreciation**

For reducing-balance depreciation the equipment’s value is reduced by the same percentage each year. So, if our machinery valued at $100 000 was depreciating at a reducing-balance rate of 10% each year, it would lose $10 000 in value during the first year (10% of $100 000) and have a remaining value of $90 000. In the second year it would lose only $9000 in value (10% of $90 000), and a progressively smaller dollar amount in each successive year.

**Book value and scrap value**

For either type of depreciation we can calculate what we call the book value of the equipment, which is the current value of the equipment. After ten years, the value of the machinery would be $(100 000 \times 0.9) \times 0.9 \times ...$, i.e. $100 000 \times 0.9^{10} = 35 000$.

We can use a similar formula to the compound interest formula, *subtracting* the value of $i$ instead of adding.

For straight-line depreciation:

*Book value = original value – $T \times$ annual depreciation*

where $T$ is the number of years.

For reducing-balance depreciation:

*Book value = original value \times (1 – i)^n*

where $n$ is the number of years and $i$ is the annual percentage depreciation rate as a decimal.
We also need to consider the scrap value of the equipment. This is a predetermined value for the equipment that, when reached, should result in its replacement.

**worked example 9**

Find the book value, after 4 years, of equipment with an original value of $250 000 if depreciation is:

(a) straight-line at 12.5% p.a. of the original value

(b) reducing-balance at 15% p.a. of the current value.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Solutions</th>
</tr>
</thead>
</table>
| (a) 1. Find the annual depreciation. | (a) Annual depreciation = \( \frac{12.5}{100} \times 250 000 \)  
= $31 250 |
| 2. Find the total depreciation by multiplying the annual depreciation by the number of years (in this case 4 years). | Total depreciation = 31 250 \times 4  
= $125 000 |
| 3. Find the book value of the equipment by subtracting the depreciation from the original value. | Book value = 250 000 − 125 000  
= $125 000 |
| (b) Use the rule: Book value = original value \( \times (1 - i)^n \). | (b) Book value = 250 000 \( \times (1 - 0.15)^4 \)  
= 250 000 \( \times 0.85^4 \)  
= $130 502 |

**worked example 10**

How many years will it take for equipment with an initial value of $400 000 to reach its scrap value of $50 000 if depreciation is reducing-balance at 17.5% p.a. of the current value?

<table>
<thead>
<tr>
<th>Steps</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Write the formula.</td>
<td>Value = original value ( \times (1 - i)^n )</td>
</tr>
<tr>
<td>2. Substitute the known values.</td>
<td>50 000 = 400 000 ( \times 0.825^n )</td>
</tr>
</tbody>
</table>
| 3. Divide both sides by the original value. | \( \frac{50 000}{400 000} = 0.825^n \)  
0.125 = 0.825^n |
| 4. Use a calculator or a spreadsheet to try different values for \( n \) until you find a satisfactory answer (in this case until 0.825^n \( \leq 0.125 \)). | \( n \) | 0.825^n
| 9 | 0.177 045 762 |
| 10 | 0.146 062 754 |
| 11 | 0.120 501 772 |
| 5. Write the answer. | It will take 11 years to reach the scrap value. |
1 Find the book value after six years (to the nearest dollar) of a fishing trawler with an original value of $300 000 if depreciation is:
(a) straight-line at 10% p.a. of the original value
(b) reducing-balance at 12% p.a. of the current value.

2 Choose the correct answer. Equipment with an original value of $125 000 is depreciated using the reducing-balance method at 12% p.a. of the current value for five years. The total amount of depreciation over that time period is closest to:
A $65 966  B $31 104  C $75 000  D $59 034  E $7500

3 How many years will it take for equipment with an initial value of $500 000 to reach its scrap value of $75 000 if depreciation is reducing-balance at 22.5% p.a. of the current value?

4 Choose the correct answer. Equipment with an initial value of $375 000 is depreciated using the reducing-balance method at 9.9% p.a. If it reaches its scrap value after 12 years, the scrap value is closest to:
A $70 500  B $107 000  C $0  D $268 000  E $37 500

5 Consider equipment with an initial value of $100 000. Find a straight-line depreciation percentage and the equivalent reducing-balance depreciation percentage that will give the same book value after five years. Your answers should be within $1000 of each other.

6 Find the difference in the book value, after six years, of a truck with an initial value of $130 000 if it is depreciated using the straight-line method at 12% p.a. of the original value, compared to using the reducing-balance method at 13% p.a. of the current value.

7 Find the reducing-value percentage that results in equipment with an original value of $200 000 having a book value of $130 000 after eight years. Give your answer correct to one decimal place.
Compound interest techniques can be used to calculate effective profit, i.e. the difference between the effective cost of an item after inflation is taken into account and the selling price.

For example, if an oil painting that cost $500 four years ago is now sold for $1200, it would appear that the owner has made a profit of $700 on the sale. However, the cost of the painting when converted to the time of sale is calculated to be $950 after taking inflation into account, so the effective profit is only $250. Often we express this effective profit as a percentage of the cost price at the time of sale. That is,

\[
\text{effective percentage profit} = \frac{\text{SP} - \text{CP}}{\text{CP}} \times 100\% 
\]

\[
= \frac{250}{950} \times 100\% 
= 26.3\%
\]

**worked example 11**

A philatelist buys a stamp for $45 and sells it for $80 six years later. If the inflation rate is 5% p.a. over that period and costs of the sale are $6, calculate the effective percentage profit in terms of:

(a) the current dollar value

(b) the dollar value at the time of purchase.

**Steps**

(a) All monies are to be converted into the equivalent amounts at time of sale.

1. Write the formula.
2. Identify \( P \), the past value of cost price.
3. Identify \( i \) and \( n \).
4. Substitute.
5. Calculate \( 1.05^6 \) on your calculator and multiply by 45. This is the equivalent cost price at time of sale.
6. Calculate effective profit.
7. Compare effective profit with cost price, converting to a percentage.

**Solutions**

(a) Current values

Cost price:

\[
A = P(1 + i)^n \\
P = 45 \\
i = 0.05, n = 6 \\
A = 45(1 + 0.05)^6 \\
= 60.30 \\
\text{CP} = 60.30
\]

Effective profit = \( \text{SP} - \text{costs} \)

\[
= 80 - (60.30 + 6) \\
= 13.70
\]

Effective % profit = \( \frac{13.70}{60.30} \times 100\% 
= 22.7\% 
\]
In Worked Example 11, the effective percentage profit is the same in parts (a) and (b), even though the effective profit in part (a)—$13.70—is different from the effective profit in part (b)—$10.22. The two amounts would have the same buying power in their respective years.
1. Assuming an inflation rate of 2.7%, calculate the effective profit or loss on a vase bought for $380 and sold nine years later for $490:
   (a) at time of sale
   (b) at time of purchase.

2. Assuming an inflation rate of 8.1%, calculate the effective profit or loss on a bottle of vintage wine bought for $80 and sold nine years later for $190:
   (a) at time of sale
   (b) at time of purchase.

3. Choose the correct answer.
   A vintage car is bought for $23,500 and sold for $30,000 seven years later. The rate of inflation during the seven-year period is 4.3%. What is the effective profit at time of sale?
   A $30,000 − $23,500(1.043)^7
   B $23,500(1.043)^7
   C $30,000
   D $30,000 − $23,500
   E $30,000 − $23,500(1.043)

4. In Question 3, what is the value of the effective profit at the time of purchase?

5. Calculate the effective percentage profit at time of sale on a coin bought for $45 and sold ten years later for $70. Assume an inflation rate of 7.2% p.a.

6. What is the effective percentage profit at time of purchase of an antique chair bought for $450 and sold for $610 in 3.5 years’ time, having been French-polished at a cost of $45? Assume an inflation rate of 6.2% p.a.

7. Calculate the effective percentage profit at time of sale on a porcelain Easter egg bought for $145 and sold ten years later for $270, if the cost of advertising was $15. Assume an inflation rate of 6.3% p.a.

8. What is the effective percentage profit at time of purchase on a Victorian dressing table bought for $2500, and sold for $3600 in 2.5 years’ time? Assume an inflation rate of 4.9% p.a. and a commission of 10% paid on the sale price.

9. What is the effective percentage profit on a first-day cover bought for $20, and sold for $41 in four years’ time? Assume an inflation rate of 5.2% p.a.

10. Find an advertisement for the car you would like to own when you first get your licence. (Be realistic!) Convert its real value to the future value of a similar car (i.e. the same age) in two years’ time, assuming 5% p.a. inflation. Calculate the amount of money you would have to save each week between now and then to buy the car outright.
Calculate the cost of each item assuming an inflation rate of 6.9% p.a., then break the code to find the caption.

N  A $378 suit in four years
T  A $2.80 hamburger in three years
U  A $510 surfboard in five years
O  A $1.50 meat pie in six years
A  A $275 jacket in four years
C  A $749 table in three years

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</thead>
<tbody>
<tr>
<td>$359.12</td>
<td>$493.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$359.12</td>
<td>$914.99</td>
<td>$914.99</td>
<td>$2.24</td>
<td>$711.97</td>
<td>$493.63</td>
<td>$3.42</td>
</tr>
<tr>
<td>$359.12</td>
<td>$493.63</td>
<td>$3.42</td>
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</table>
Many people are interested in making money. One way you can do this is to invest any ‘spare’ money you may have, but how effective is this?

Let us assume you have $10 000 available for investment. You are able to invest this for a three-year term at a fixed interest rate of 8% p.a. compounded annually.

In your answers to the following questions, give money answers to the nearest cent and other answers correct to two decimal places.

1. How many dollars will be in your account at the end of the three-year term?

At the end of the three-year term you are required to pay income tax on the interest you have earned.

2. Assuming the tax rate is 47%, calculate the amount of tax you have to pay.

Over the course of the three-year period, inflation has averaged 4% p.a.

3. How many dollars do you need to have after paying tax and taking account of inflation not to have lost money on your investment?

4. So, how have you fared as an investor? Did you make a profit or a loss? Express this profit or loss as a percentage (work in current dollar terms).

5. What would the average inflation rate need to be so that your investment broke even—i.e. made neither a profit nor a loss?
Summary

- Household bills are often calculated by multiplying the number of units by a rate. GST is added after the charges are calculated.
- Budgets involve finding equivalent amounts of money for different intervals of time.
- Simple interest formulae: \( I = PRT \) \( A = P + I \)
- Compound interest may be worked as a series of simple interest calculations, with changing principal values if the interest rate changes, or if a spreadsheet is being used.
- Loans (especially house loans) involve interest charged on reducing balance.
- Compound interest formula: \( A = P(1 + i)^n \)
- Depreciation:
  - Straight-line: Book value = original value \(-\) T \times annual depreciation
  - Reducing-balance: Book value = original value \times (1 - i)^n
- When inflation is taken into account:
  - effective profit at time of sale and at time of purchase can be calculated;
  - \( \text{effective percentage profit} = \frac{\text{effective profit at time of sale}}{\text{value of cost price at time of sale}} \times 100\% \)

FAQs

What is the difference between simple interest and compound interest?
With simple interest the same amount of interest is earned (or owed) for each year or other period. With compound interest the amount of interest earned (or owed) is added to the principal for the next interest period, making a difference in the amount earned (or owed).

How do you adjust a price to take account of inflation?
You need to know the average inflation rate over the period of time in question. You then apply the compound interest formula.

Skills

1. A family has the following ongoing expenses:
   - Rent—$205 per week
   - Electricity—$480 per 90 days
   - Medical insurance—$1560 per annum
   - Newspapers—$48 per month
   - Telephone—$112 per month

   Calculate (to the nearest dollar) the total amount that needs to be budgeted to cover these costs:
   (a) weekly
   (b) monthly.
2 Calculate the total amount owing on a loan of $8000 after two years if the interest is 12% compounded annually.

3 Choose the correct answer.
   The interest on a $15 000 loan at 4.2% p.a. compounded quarterly for four years, with no repayments made, is closest to:
   A $211  B $2739  C $2728.50  D $640  E $274

4 Scott deposits $20 000 towards the cost of a mobile home he plans to buy in six years’ time. Find the interest earned if it is calculated at:
   (a) 6.9% simple interest  (b) 6.5% p.a. compounded annually
   (c) 6.25% p.a. compounded quarterly  (d) 6% compounded monthly.

5 Choose the correct answer.
   Equipment with an original value of $95 000 is depreciated using the reducing-balance method at 15% p.a. of the current value for five years. The book value at the end of the five years is closest to:
   A $71 250  B $42 152  C $23 750  D $52 848  E $62 418

Applications

6 The following charges apply for long distance calls over various distances.

<table>
<thead>
<tr>
<th>Call type</th>
<th>Connection fee (cents)</th>
<th>Charge (cents per second)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Day rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 a.m.–6 p.m.</td>
</tr>
<tr>
<td>NDD2</td>
<td>12.0</td>
<td>0.20465</td>
</tr>
<tr>
<td>NDD3</td>
<td>12.0</td>
<td>0.43483</td>
</tr>
<tr>
<td>NDD4</td>
<td>12.0</td>
<td>0.44950</td>
</tr>
</tbody>
</table>

Find the cost of each of these calls (in dollars).
   (a) an NDD2 call for 200 seconds at 7 p.m. Friday
   (b) an NDD4 call for 2 1/2 minutes at 9 a.m. Monday
   (c) an NDD3 call for 6 minutes at 9 p.m. Thursday
   (d) an NDD2 call for 7 1/4 minutes at 3 p.m. Monday
   (e) an NDD3 call for 900 seconds at 6.30 p.m. Sunday

7 Use the repayments table on page 13 to help with this question.
   (a) Find the interest cost of a housing loan for $190 000 at:
       (i) 5% for 5 years  (ii) 5% for 10 years  (iii) 5% for 15 years.
   (b) In each case in part (a), find the percentage of the total repayments that is interest. Give your answers correct to two decimal places.
   (c) Repeat parts (a) and (b) using an interest rate of 6%.
8 (a) How much would Caitlin need to invest now to receive $15 200 in four years’ time if she puts her money in an account that pays 2.9% p.a. interest compounded monthly? Answer to the nearest dollar.

(b) After how many years will a block of land currently valued at $70 000 be worth at least $100 000 if it increases in value by 7.5% p.a.?

9 Find the reducing-balance percentage that results in a truck with an original value of $235 000 having a book value of $150 000 after four years. Give your answer correct to one decimal place.

10 What is the effective percentage profit on an Edwardian dining suite, bought for $2800 and sold for $4600 in 3.5 years’ time? Assume inflation of 4.9% p.a. and a commission of 10% paid on the sale price.

11 What is the effective percentage profit on a ruby bought for $500 and sold for $1000 in five years’ time? Assume an inflation rate of 3.2% p.a.

12 A block of land is purchased for $48 000. It is sold four years later for $84 000. At the end of each of the four years council rates of $200 are paid, as well as maintenance costs of $300. There is a 10% commission to the selling agent. If inflation has averaged 3.5% p.a. over the course of the four years, find the profit (or loss) made on the property in terms of the value of the dollar at the time of sale.

1 Expand:
(a) \(4(m + 5)\)  
(b) \(a(a - 7)\)  
(c) \((x + y)(x - y)\)

2 Simplify:
(a) \(\sqrt{18}\)  
(b) \(\sqrt{24}\)  
(c) \(3 \sqrt{200}\)

3 Express in standard form.
(a) \(657000\)  
(b) \(0.0105\)

4 Calculate, expressing your answer in scientific notation, correct to two decimal places.
(a) \(5.6 \times 10^6 \times 4.2 \times 10^3\)  
(b) \(\frac{4.3 \times 10^7}{2.1 \times 10^4}\)  
(c) \((5.4 \times 10^5)^3\)

5 Calculate, showing full working.
(a) \(\frac{1}{2} + \frac{3}{4}\)  
(b) \(\frac{1}{2} - \frac{3}{8}\)  
(c) \(\frac{2}{3} \times \frac{5}{6}\)  
(d) \(\frac{3}{4} + 2\)

6 Calculate, showing full working.
(a) \(27.01 + 5.3\)  
(b) \(1000 - 28.1\)  
(c) \(432.7 \times 2.6\)  
(d) \(873 + 7\)
7 Convert:
(a) 93 cm to m  (b) 40 mL to L  (c) 600 kL to m³
(d) 4000 m² to ha  (e) 75 cm³ to mL  (f) 6500 mm to m

8 Find the perimeter and area of each of these shapes, correct to one decimal place where appropriate
(a)  (b)  (c)

9 Find the surface area and volume of each of these solids.
(a)  (b)  (c)

10 Find the values of the pronumerals correct to one decimal place.
(a)  (b)  (c)

11 Factorise:
(a) 6x + 8  (b) 15p − 3p²  (c) 7m − mn − 4n + 28
(d) 9 − x²  (e) x² + 16x + 64  (f) x² − 15x + 50

12 Solve:
(a) 2x + 5 = 17  (b) 2y − 8 = 5 − 4y  (c) \( \frac{3x - 2}{7} = 1 \)
(d) 3m − 5 = 13  (e) 6 − 5a = 1  (f) x² = x + 2