amusement park physics

Key Concepts

F1, F2, F4, E1, E2, E3, E4, M1, M2

Key Ideas

- Acceleration
- Circular motion
- Energy and work
- Equations of motion
- Friction
- Gravity
- Hooke's law
- Kinetic energy
- Mass and weight
- Newton's first law of motion
- Newton's second law of motion
- Newton's third law of motion
- Pendulum
- Potential energy
- Power—mechanical
- Vectors
- Velocity

Related Key Ideas

- Simple harmonic motion

figure ap.1 Amusement park rides depend on physics for their safe construction.

Wouldn't it be great to go to a fun park like Dreamworld or Disneyland for every physics class? Where every lesson was about something there: a ride, a machine, a drink dispenser. Well, without the application of physics principles there would be very little fun in a fun park. Even sliding down a slippery slide at the local park involves balancing gravitational and frictional forces. During this context we will investigate the applications of physics in an amusement park or theme park, allowing you to analyse existing facilities or design new ones. You'll gain a better understanding of the tremendous effort that has gone into the design of even fairly simple rides.

Ferris wheels

The force experienced by riders on a Ferris wheel is the result of two forces acting on their bodies. The force of gravity will always be there. There is also the force being applied by the seat — it is this force that stops the rider falling, and makes them move in a circle.

* See Circular motion on page 350. See Gravity on page 395.

figure ap.2 A Ferris wheel.

Since the riders move in a circle, there must be a centripetal force which will remain constant (since the path of motion is a circle). The centripetal force is the result of adding the force of gravity and the force supplied by the seat.

$$\boldsymbol{F}_{\text{centripetal}} = \boldsymbol{F}_{\text{gravity}} + \boldsymbol{F}_{\text{seat}}$$

Since the centripetal force must always be towards the centre of the circle of motion (that is, towards the axle of the Ferris wheel), and the force of gravity is always downwards, then the force being supplied by the seat must change in direction and size.

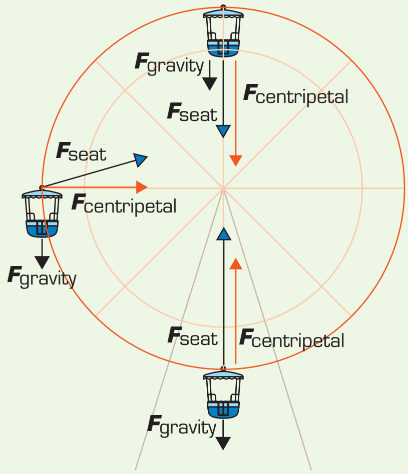


figure ap.3 The forces acting at various points in a Ferris-wheel ride. Note that the centripetal force is constant.

Using the centripetal force equation and Newton's second law:

$$\boldsymbol{F}_{\text{centripetal}} = \boldsymbol{F}_{\text{gravity}} + \boldsymbol{F}_{\text{seat}}$$

$$\frac{m\boldsymbol{v}^2}{r} = m\boldsymbol{g} + \boldsymbol{F}_{\text{seat}}$$

This is a vector sum, so the angles between these forces are important. At the top and the bottom of the ride, the forces are acting vertically (see figure ap.3), so the maths is simple. At the sides of the ride, trigonometry will need to be used to determine the value of the forces.

For a given size of Ferris wheel, it is easy to calculate the maximum speed at which it should turn, for comfort and safety. The total force experienced by the riders is the centripetal force, which is the vector sum of the force exerted by the seat and the gravitational force. Although the force exerted by the seat changes magnitude and direction, the riders only experience the resultant centripetal force. This has constant magnitude but variable direction (relative to the rider; the absolute direction is always towards the centre, but this might be 'up', 'down' or at an angle relative to the rider).

$$\boldsymbol{F}_{\text{centripetal}} = \frac{m\boldsymbol{v}^2}{r} \Rightarrow \boldsymbol{v}_{\text{max}} = \sqrt{\frac{\boldsymbol{F}_{\text{max}}r}{m}}$$

$$\boldsymbol{a}_{\text{centripetal}} = \frac{\boldsymbol{v}^2}{r} \Rightarrow \boldsymbol{v}_{\text{max}} = \sqrt{\boldsymbol{a}_{\text{max}}r}$$

The effects of acceleration on a rider are discussed later in this context.

Turning speeds are often expressed in revolutions per minute (rpm). The conversion is fairly simple. Ignoring the direction of travel, the velocity becomes a speed.

$$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r \times \text{rpm}}{60\ \text{s min}^{-1}}\ \text{m s}^{-1}$$

$$\Rightarrow \text{rpm} = \frac{60 \times v}{2\pi r}$$

Exercises

E1 If the riders are not to experience more than 10 m s^{-2} acceleration, what is the maximum safe velocity of a Ferris wheel with a radius of 9.0 m?

E2 What is the force experienced by a 45 kg rider on a Ferris wheel of radius 25 m which is turning at 5.0 m s^{-1}?

E3 What is the acceleration experienced by a 30 kg child riding a Ferris wheel that has a diameter of 20 m and is turning at 5.0 m s^{-1}?

E4 If a Ferris wheel of diameter 10 m is being designed for very small children, who should not be subjected to accelerations greater than 0.2$\boldsymbol{g}$, what is the maximum speed at which it should turn?

E5 What is the speed of the Ferris wheel in the question E4 in rpm?

E6 At what maximum rpm should a Ferris wheel of diameter 30 m turn if the riders are not to experience an acceleration of more than 0.5$\boldsymbol{g}$?

Roller coasters

A roller coaster car depends on the action of gravity for movement over most of its journey. The car is hauled to the highest point of the track, and then released, to roll down to the bottom. Along the way, bumps, loops, and turns make the ride more exciting. The length of the ride will depend on two factors: the height to which the car is hauled at the start, and the friction between the car and the track.

figure ap.4 A roller coaster relies on the force of gravity for its exhilaration.

See Friction on page 392. See Gravity on page 395.

Rolling down a hill

Let's start by looking at a very simple ride. The car is hauled to the top of a hill, and rolls down a straight track to the bottom. At the top of the hill, the stationary car has gravitational potential energy $E_p = mgh$, where m is the mass of the car (kg), $\boldsymbol{g}$ is the gravitational acceleration (9.8 m s^{-2}), and h is the height (m). The height is with respect to some reference point, typically the surface of the ground underneath the car. Any reference point can be chosen as long as all calculations within a situation relate to the same reference point. It simplifies calculations to choose a reference point so that the gravitational potential energy at some part of the situation is zero and the rest is positive; that is, choose a reference point at the lowest part of the track. In this case, choosing a reference point at the lowest point of the track allows us to ignore the gravitational potential energy at that point, and therefore the height in the equation is the height above the lowest point of the track.

See Potential energy on page 509.

As the car starts to move down the slope, it will accelerate due to the force of gravity. The vertical acceleration will be $\boldsymbol{g}$, down, but since the car is not falling vertically, it will gain horizontal and vertical velocities that can be determined by the equations of motion, once we know the elapsed time and the angle of the slope away from the vertical θ.

See Equations of motion on page 384.

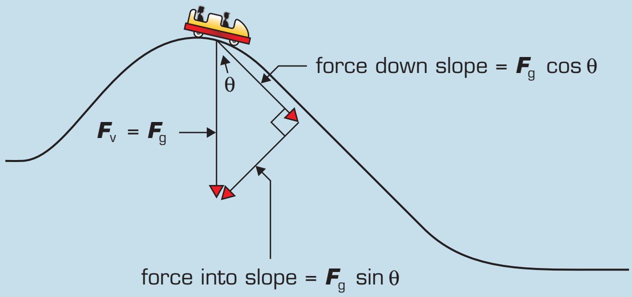


figure ap.5 The down-slope and into-slope components of the force acting on a roller coaster car travelling down the slope.

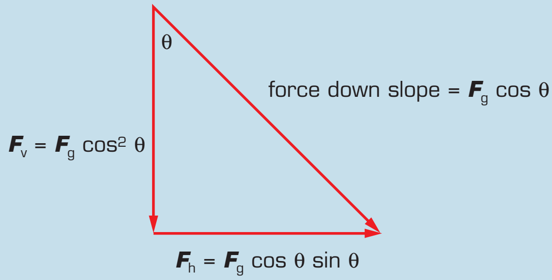


figure ap.6 The vertical and horizontal components of the force down the slope.

Down the slope, the force and acceleration can be calculated by:

$$F_{g,\text{ down slope}} = F_g \cos\theta = mg\cos\theta$$

Substituting into $a = \dfrac{F}{m}$:

$$\Rightarrow a_{g,\text{ down slope}} = \frac{mg\cos\theta}{m} = g\cos\theta$$

We know $v = u + at$ so:

$$v_{\text{down slope}} = u + at = gt\cos\theta$$

Similarly, into the slope:

$$F_{g,\text{ into slope}} = mg\sin\theta$$

The force into the slope is countered by the track; otherwise the car would sink into or fall through the track. The force into the track and the force of the track pushing back are equal and opposite, and therefore cancel each other out, resulting in no acceleration in that direction.

The force down the slope can be resolved into horizontal and vertical components. Note that this is not a reversal of the above procedure, as the right angle of the vector triangle is in a different position. As a check on validity, it should be seen that the force down the slope should be smaller than the gravitational force (unless the track is vertical, in which case they are equal). Also, the vertical component of the force down the slope should be less than the force down the slope.

See Vectors on page 563.

The vertical force and horizontal force are given by:

$$F_{\text{vertical}} = F_g\cos^2\theta$$
$$F_{\text{horizontal}} = F_g\cos\theta\sin\theta$$

Given a time interval, and the angle of the slope, and the length of the slope, it is possible to calculate the fastest speed that the car will reach, but there is an easier way. By using the principle of conservation of energy (which states that energy cannot be created or destroyed), we can say that:

$$E_{p,\text{ at top of ride}} = E_{k,\text{ at bottom}} + E_{\text{lost through friction, sound, heat, wear}}$$

See Energy and work on page 379.

Calculating the energy lost through friction from a theoretical perspective is very difficult, but it will be shown later that it is very easy to measure (see the following section '... and up the other side'). We can calculate the kinetic energy, $E_k = \frac{1}{2}mv^2$, so, for the time being, ignoring the energy losses from friction, we can say that:

$$E_p = E_k$$
$$mgh = \frac{1}{2}mv^2$$

and since the masses cancel,

$$gh = \frac{1}{2}v^2$$
$$v^2 = 2gh$$
$$v = \sqrt{2gh}$$

Actually, $v^2 = 2gh$ is just a special case of one of the equations of motion: $v^2 = u^2 + 2as$, where $u = 0$, $a = g$, and $s = h$.

See Equations of motion on page 384.
See Kinetic energy on page 419.

This velocity represents the greatest velocity possible for an object dropped from a height, h. Interestingly, it is independent of mass. The gravitational force acting on a larger mass is greater, but the greater mass requires a larger force to achieve the same acceleration, so these two effects cancel each other out. The exact path (the slope or shape of the track) does not seem to matter either. Where the shape and slope of the pathway will be important is in considering how much friction is encountered. Friction will slow down the object, so more friction because of a longer path should mean a slower car at the end of the ride.

* See Friction on page 392. See Velocity on page 570.

Worked example ap.1

What is the theoretical maximum speed of a roller coaster car starting at a height of 75 m above the lowest point of its track?

Solution

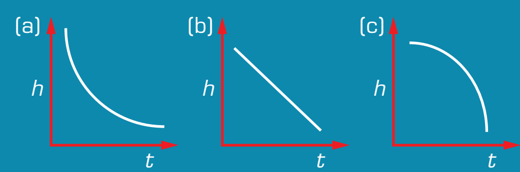
We have $\boldsymbol{g} = 9.8\ \text{m s}^{-2}$, and $h = 75$ m.

$$\boldsymbol{v} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 75} = 38.3$$

Hence, the theoretical maximum speed is $38.3\ \text{m s}^{-1}$.

Experimental investigation

There are three general types of paths possible for the roller coaster: (a) exponential (dropping steeply and then levelling off), (b) a straight line, and (c) another exponential (dropping slowly, and then steeply). These have different acceleration profiles and result in different time intervals to get to the same point. Obviously, the straight line is the shortest distance to travel, but it is *not* the *quickest* way down! See if you can figure out which will be the quickest, and why. Confirm your answer with toy cars and plastic track. Time how long it takes a toy car to reach a certain point, not how long it takes to travel the length of a section of track.



The equations show that the greatest possible velocity depends on the height from which the object starts dropping or rolling. Therefore, we have found that the higher the start of the ride, the faster it will be going at the bottom. Sometimes it is good to realise that physics can prove something that is so obviously true!

Theoretically then, our well-oiled roller coaster track can have as many loops, bumps, and turns as we like without affecting how fast the roller coaster car will be going at the bottom. Realistically, though, the shape and length of the track will affect the duration and size of the frictional force, which will slow the car down. Therefore loops, bumps and turns will all act to slow the car down, and make for a less interesting speed thrill. A straight drop down gives a better speed thrill, but has less interest. A reasonable compromise would be to drop the car from the top to the lowest point straight away, to gain the maximum speed thrill, and then go into the turns and loops. This large drop before the turns and bumps is a common design feature of roller coasters. Look for it next time you see one.

… and up the other side

Let's make the track a little more complicated: the shape of a U. How far will the car go up the other side? The kinetic energy at the bottom of the U is equal to the potential energy that the car had at the start (at the top of the U). Since energy cannot be created or destroyed, the kinetic energy gets converted back into gravitational potential energy as the car moves up the other side and slows down. The height that the car reaches on the other side *should* equal the height it started from, but it won't. The reason for this is that some energy is lost to the system through friction: energy is being given out as heat or sound, or wear. Think about the rumbling sound a roller coaster makes; a large amount of energy is being wasted to produce that sound.
This gives us a way to measure the energy lost through friction. Since:

$$E_{\text{p, at top of ride}} = E_{\text{k}} + E_{\text{lost through friction, sound, heat, wear}}$$
$$E_{\text{p, at top of ride}} = E_{\text{p, at high point of other side}} + E_{\text{lost through friction, sound, heat, wear}}$$

Then:

$$\begin{aligned} E_{\text{lost through friction}} &= E_{\text{p, at top of ride}} + E_{\text{p, at high point of other side}} \\ &= m\boldsymbol{g}\Delta h \end{aligned}$$

where m is the mass of the object (kg), $\boldsymbol{g}$ is gravitational acceleration (9.8 m s^{-2} on the surface of the Earth), and Δh is the measured height difference (m).

This gives us a way of measuring the energy lost through friction and might be of interest, but it is not of practical use in designing a roller coaster, since it can only be used after the coaster is built. Understand, though, that a roller coaster ride can have hills and bumps, but each high point encountered on the ride must be lower than the previous high point to compensate for the energy loss due to friction. The calculations of theoretical frictional energy loss are too complex to investigate here. In your own design of a model roller coaster, make what you think is a reasonable adjustment for friction by lowering the maximum heights by a certain percentage each time. If you are wrong in your guess, it is not too difficult to adjust a model (at least compared with a real coaster!)

Worked example ap.2

A 500 kg roller coaster at a fairground starts 50 m above the ground, goes down into a dip, and just manages to roll over the next hill that is 42 m above the ground. How much energy has been lost through friction?

Solution

We know that $m = 500$ kg, $\boldsymbol{g} = 9.8$ m s^{-2}, and $\Delta h = 50 - 42 = 8$ m.

$$\begin{aligned} E_{\text{lost through friction}} &= E_{\text{p, at top of ride}} + E_{\text{p, at high point of other side}} \\ &= m\boldsymbol{g}\Delta h \\ &= 500 \times 9.8 \times 8 = 39\ 200 \end{aligned}$$

Hence, the energy lost through friction is 39 200 J or 39.2 kJ.

Experimental investigation

A roller coaster rolling backwards and forwards through a U-shaped section of track appears to have very similar motion to that of a pendulum. Design and conduct an experiment to determine whether or not a pendulum and a coaster car move in the same way.

Exercises

E7 If a roller coaster car has a mass of 1.0 tonne including the passengers, what force will be accelerating it down a track that is 40° away from vertical?

E8 If the gravitational force on a roller coaster car is 500 N when acting down a slope that is 30° below horizontal, what is the mass of the car?

E9 What is the theoretical maximum speed of a roller coaster car starting at a height of 50 m above the lowest point of its track?

E10 A 900 kg roller coaster car starts its run 75 m above the ground, and rolls to a height of 62 m up a smooth U-shaped track before slowing to a stop. How much energy has been lost due to friction?

E11 The owner of a fairground wants a roller coaster that will travel at 100 km h^{-1}. How high will it need to be?

E12 Will you travel faster, slower, or at the same maximum speed if you are the only person on a roller coaster ride, compared with sharing the ride with nine other people? (Do not ignore the effect of friction.)

E13 A roller coaster car is approaching a rising slope at 19 m s^{-1}. How far can it travel up the slope, assuming no energy is lost through friction?

E14 Assume that each energy transformation (e.g. kinetic → potential) loses 10% to friction. (This is an oversimplified, non-realistic assumption.) On a sheet of graph paper, accurately represent a sideways view of a roller coaster track designed so that each successive hill is at the maximum height to allow the coaster car to just roll over it.

Acceleration is fun?

The excitement of a roller coaster is also due to the unusual accelerations experienced during the ride, producing apparent forces from weightlessness to increased weight, and being thrown sideways. These accelerations have effects on the human body, depending on the direction that they act in.

* See Acceleration on page 319. See Mass and weight on page 463.

A person's blood is only loosely held in the body; think of the body as a bag of water. When the body is accelerated upwards, the blood will tend to stay where it is (as defined by Newton's first law on inertia). Vertical acceleration upwards can therefore cause the blood in a person's body to drain away from their brain and into their feet. This can cause a blackout (a temporary loss of consciousness).

* See Newton's first law of motion on page 483.

Vertical acceleration downward (not just falling, because the blood will accelerate down at the same time as the body during a fall, but, for example, during a power dive in a plane) causes the opposite effect: the brain becomes oversupplied with blood, and a 'red-out' can also result in loss of consciousness. A person may also vomit when experiencing large downward accelerations because the stomach contents will also tend to stay in place due to inertia, as their body accelerates downwards.

If vertical acceleration in either direction is continued for a long time, death can result because a 'red-out' can cause bursting of blood vessels in the brain and a 'blackout' can allow brain cells to die due to lack of oxygen. Some people are more susceptible to these effects of acceleration than others. The NASA astronaut training program tests people in special machines to see if they can tolerate these extremes of acceleration before allowing them to continue training.

The human body is better able to cope with horizontal acceleration, where the blood flow up and down the body is not affected. Up to 4g (where g is the gravitational acceleration, 9.8 m s^{-2}) can be tolerated by most people, and up to 10g can be tolerated for very brief periods. For comparison purposes, a car accelerating from 0 to 100 km h^{-1} in 6 s is accelerating at 0.47g. To accelerate at 4g, a car would go from 0 to 100 km h^{-1} in just 0.7 s!

In either horizontal or vertical acceleration, people who are not healthy are more likely to suffer

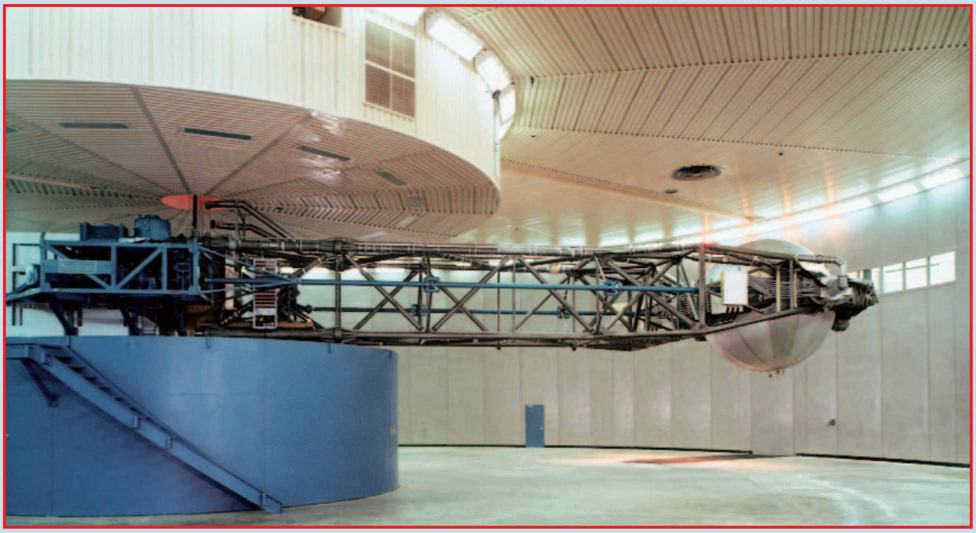


figure ap.7 Astronauts are tested in a ***g***-force simulator before being allowed to go on space missions. The astronaut is seated in the chamber at the end of the spinning arm.

ill-effects. There is also a risk with sudden accelerations of falling against an object, or even suffering whiplash, where an injury occurs to the neck because the neck muscles cannot hold the head upright against the forces involved. If you have ever stood in a swerving bus, you would be aware that even very low accelerations (typically less than 0.1*g* in a bus) could cause you to lose balance and stumble or fall. Imagine what it would be like if the acceleration were 100 times greater!

Force is a vector, so the resultant force felt by the person on the roller coaster is the vector sum of all the forces on the person. Of course, one of these is always the gravitational force, which operates down, and is equal to 9.8 N kg^{-1}. To this we can add whatever other forces are being experienced by the car.

* See Newton's second law of motion on page 486.
See Vectors on page 563.

The person feels weight because the gravitational force pulls down, but the material that they are standing on resists any motion in that direction. It effectively pushes back up. That upward push is usually equal and opposite to gravity, so the two cancel each other out, and there is no resultant vertical acceleration or motion. It is this upward push that gives the sensation of weight, even though the gravitational force is downwards.

* See Newton's third law of motion on page 488.

In free-fall, the situation is a little more complicated. The person feels weightless. (Note that there is still gravity; it is never correct to say that there is no gravity.) You have probably felt a similar 'loss of weight' when standing in an elevator as it starts to go down. There is still the force of gravity when free-falling, but since there is nothing opposing it, there is no resistance, and no *sensation* of weight.

This feeling of weightlessness can cause severe nausea in some people, so free-falling is not recommended for all rides. On the roller coaster, it is not likely that the design will permit a true free-fall situation because of the action of friction. In any situation where the car is rolling down a slope, no matter how steep (as long as it is not vertical), there will be some friction between the car and the track. There will also be air resistance. This will reduce the apparent force of gravity to less than 9.8 N kg^{-1}, because the friction is acting in an upward direction. Even if the frictional force is very small, unless the track is vertical, the force of friction must be resolved into horizontal and vertical components, and the vertical resultant will therefore be less than the force of gravity.

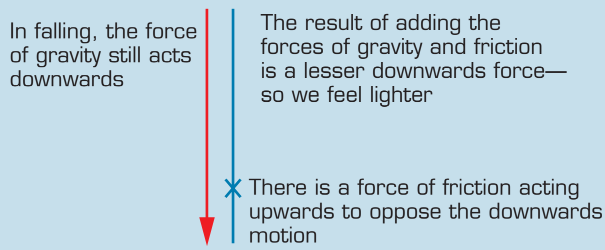


figure ap.8 When accelerating vertically we can gain a sense of reduced weight because the force of friction acts against the force of gravity, and the resultant downwards force is less than the force of gravity alone.

Therefore, as the car starts to drop, the riders will feel less heavy as they accelerate downwards. This explains the stomach-in-the-mouth feeling as the roller coaster begins to drop, typically resulting in screams of terror. (We learn at a young age that falling is painful, so the sensation of falling produces a fear response.)

When the car reaches the bottom of a dip in the track, and starts moving up the other side, it is accelerating downwards while moving upwards. The *downwards* acceleration is due to the force of gravity, and will slow the car down as it goes up the slope on the other side. However, since the direction of movement (vertical velocity) of the car has changed from downwards to upwards there has also been an *upwards* acceleration of the car, and therefore there must be a force acting upwards. This force is supplied by the track. Since the car is therefore accelerating upwards due to the direction change, the people in the car must also accelerate upwards against their inertia. The changing inertia will be experienced as though it is a force. In the roller-coaster car, the riders experience this as a force produced by the car pushing them upwards. This force is added to the force of gravity, and so the riders feel heavier.

Worked example ap.3

A roller-coaster car is approaching the top of a hill in the track, 15 m lower than the summit. It is travelling at 10 m s^{-1} vertically up. Another 4 s later, it is travelling at 12 m s^{-1} vertically down and is 15 m down the other side of the hill. What average acceleration do the riders experience?

Solution

If we take down as the positive direction, the change in velocity is:

$$\begin{aligned}\Delta \boldsymbol{v} = \boldsymbol{v} - \boldsymbol{u} &= 10 \text{ m s}^{-1} \text{ down} - 10 \text{ m s}^{-1} \text{ up}\\ &= -10 \text{ m s}^{-1} - 10 \text{ m s}^{-1} = -20 \text{ m s}^{-1}\end{aligned}$$

And $t = 4$ s.

$$\boldsymbol{a} = \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{-20}{4} = -5$$

Hence, the riders experience an average acceleration of 5 m s^{-2} downwards.

An alternating series of bumps and dips (hills and valleys) will cause the riders to experience alternate sensations of lightness and heaviness. The degree of sensation will depend on the vertical acceleration, which will depend on the steepness of the track. The change in direction from upwards to downwards, or vice versa, divided by the time it takes to do so, is equal to the acceleration experienced. A shorter time interval will result in greater accelerations. Having steeper hills or valleys in the track can create a shorter time interval, and greater accelerations. Therefore, a compact track design will result in more intense sensations than a long gently sloping one.

Loops

Going through a loop is not all that different from going over a hill the right way up, except for the minor difference of being upside-down!

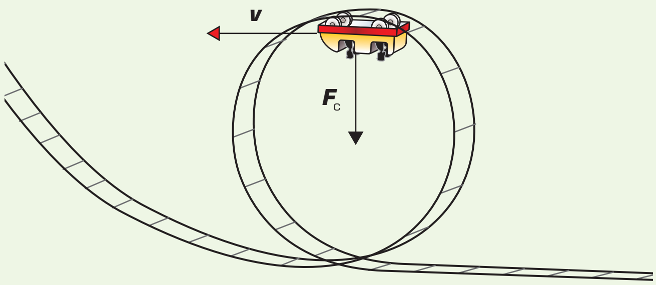


figure ap.9 A roller-coaster car going through a loop.

The centripetal force is supplied by the track 'pushing' the car around and, when the car is at the top of the loop, by the gravitational force too. Actually, it is more correct to say that the structure of the track is preventing the car from travelling in a straight line, rather than 'pushing', but the effect is the same.

See Circular motion on page 350.

We experience weight because the ground resists our falling. That is, gravity pulls us down, the ground pushes back and we 'feel' weight. In a free-fall situation, gravity still pulls us down, but we experience weightlessness. In the roller coaster, the riders are going to experience the centripetal force supplied by the seat as a push, and incorrectly interpret it as being 'thrown' upward.

During the journey through the loop, the car and rider are subjected to a centripetal force, $\boldsymbol{F}_c$, that makes them travel in a circle. Assuming that the loop is circular, the force on the rider will be constant, but it is made up of the vector sum of two other forces: $\boldsymbol{F}_g$, and the force exerted by the seat, $\boldsymbol{F}_s$. $\boldsymbol{F}_c$ will always point towards the centre of curvature of that part of the track. $\boldsymbol{F}_g$ will always act downwards. $\boldsymbol{F}_s$ will act in a direction such that the vector sum $\boldsymbol{F}_g + \boldsymbol{F}_s$ will equal $\boldsymbol{F}_c$.

See Vectors on page 563.

The riders will notice a difference in the force being exerted on them by the seat and seat belt as they go through the loop. Since $\boldsymbol{F}_c = \boldsymbol{F}_g + \boldsymbol{F}_s$, then $\boldsymbol{F}_s = \boldsymbol{F}_c - \boldsymbol{F}_g$; $\boldsymbol{F}_s$ must make up any difference between $\boldsymbol{F}_g$ and the centripetal force needed to make the rider travel in a curve. This gives four alternatives: $\boldsymbol{F}_s$ might act upwards (riders feel they are being held upside down), or downwards (riders feel they are being thrown upward), or it might be zero (riders feel weightless) depending on the value of $\boldsymbol{F}_c - \boldsymbol{F}_g$.

The fourth alternative is that $F_s = mg$ and the riders feel the same weight as they usually feel.

Worked example ap.4

On a roller-coaster loop with a radius of 20 m, what speed would the car have to travel at as it enters the loop to make sure that it goes over the top? Ignore friction.

Solution

We have $g = 9.8$ m s^{-2}, and $h = 2 \times 20 = 40$ m, double the radius of the roller coaster. Kinetic energy as it enters the loop becomes gravitational potential energy at the top of the loop, so:

$$v_{min} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 40} = 28$$

Hence, the speed at which the car would have to travel is 28 m s^{-1}.

figure ap.10 A roller coaster going through a banked turn.

At the bottom of the loop, the riders experience the same sensation as at the bottom of a valley. The centripetal force acts upwards, causing the riders to experience greater weight. The amount of weight a rider 'feels' is given by:

$$F_{\text{seat, at top}} = F_g - F_c$$
$$F_{\text{seat, at bottom}} = F_g + F_c$$

During the journey from the bottom of the loop to the top, kinetic energy is being converted to gravitational potential energy, so we can use the equations derived in the section on roller coasters to calculate the maximum speed needed to reach the top section of the loop. The fact that the car is upside down has only a minor effect in that the radius of the loop is slightly less because the roller coaster car is inside the track instead of outside it.

Frictional forces are still present as long as the car stays in contact with the track, but are too difficult to predict theoretically.

Turns

When you are in a motor car speeding around a corner, you feel as though you are being thrown to one side. Actually, you are experiencing something far more complicated, since there is no force acting to push you out. (If this answer surprises you, think about what is doing the pushing, and remember that 'moving' and 'inertia' are *not* forces.)

Let us look at a car turning a corner. Your inertia is straight ahead. The action of the car tyres on the road is to turn the car. Your body tends to continue moving straight ahead, but is pushed sideways by the car, its seat belts and so on. Because you can feel the pressure of the car pushing you sideways, it *feels* like you are being thrown against the side of the car. In actual fact, the car is pushing against your side, forcing you to go around the corner!

The size of the force can be determined from the centripetal acceleration equation:

$$a_c = \frac{v^2}{r}$$

$$F = ma$$

$$\Rightarrow F_c = \frac{mv^2}{r}$$

In this case, the size of the force experienced by a person will depend on that person's mass, the speed at which the car turns the corner and the radius of turning. Obviously, higher speeds and sharper curves result in greater accelerations and greater forces.

* See Circular motion on page 350.

The force experienced as a force exerted by the seat or side of the car is mass-dependent; that is, a heavier person experiences a greater force. The greater force acting on a greater mass results in the same acceleration, so it is usual to discuss the acceleration (in gs) experienced by the riders as this is independent of mass.

Twist and turn, and shake it all about

The analysis of a complex or corkscrew manoeuvre on a roller coaster can be quite intimidating. Remember though that the resultant force experienced by the riders is merely the sum of all the forces acting at the time. Therefore, in a combination turn and loop, like a corkscrew or a tilted loop, the resultant force experienced is the vector sum of the circular motion effect caused by the horizontal turn, and the circular motion effect caused by the vertical loop. The only difficulty is that the analysis can involve three-dimensional resolution of all of the vectors if you choose the wrong frame of reference. Calculations are a little easier if you can choose a frame of reference that includes most of the forces and directions of travel.

figure ap.11 Combinations of loops and turns provide even more excitement!

Exercises

The following information applies to exercises E15 and E16. A roller coaster car is approaching the top of a hill in the track which is 10 m lower than the summit. It is travelling at 10 m s^{-1} vertically up. Another 2 s later, it is travelling at 12 m s^{-1} vertically down and is 12 m down the other side of the hill.

E15 What average acceleration do the riders experience?

E16 One of the passengers in the car has a mass of 80 kg.
- a How heavy will this passenger feel during the ride over the hill?
- b If the roller coaster were going through a valley of the same shape, how heavy would the passenger feel then?

E17 On a theoretical roller coaster, the car maintains a constant speed of 20 m s^{-1} as it travels through the bottom of a U-shaped section of track which has a length of 40 m. The entry slope has an angle of 20° off vertical, and the exit slope is 40° off vertical. What average acceleration will the passengers in the car experience if it takes 3 s to go through this section?

E18 A roller-coaster car starts at the top of a track and travels down a 45° angle into a U-shaped valley of radius 30 m. What is the maximum safe height for the top of the track if the design requires accelerations of no more than 1$\boldsymbol{g}$?

E19 A designer wants to build the world's largest loop, with a radius of 100 m. Ignoring friction, what speed would the car have to travel at as it enters the loop to ensure that it makes it to the top of the loop?

E20 Will objects drop out of the pocket of a rider on a roller coaster that goes upside down through a loop? Explain your reasoning.

E21 As a roller-coaster car reaches a curve, it is travelling at 6.0 m s^{-1}. If the designers of the ride want the horizontal acceleration felt by the passengers to be less than 2$\boldsymbol{g}$, what is the minimum radius curve that could be placed at this location?

E22 To fit into the available space, a curve on a roller-coaster track has a radius of no more than 10 m. At what maximum speed can the car enter the curve if the acceleration felt by the riders is not to exceed 1.5$\boldsymbol{g}$?

E23 A roller-coaster car with its maximum load of passengers weighs 1 tonne. If it travels at 15 m s^{-1} into a curve of radius 20 m, what force must the track be able to withstand?

E24 Because of track and roller-coaster car limitations, the designers cannot have a sideways acceleration of more than 1*g* on a particular ride. How can the designers still create a ride in which the passengers experience 2*g*, without redesigning the car or track?

Power requirements for a roller coaster

Since the roller coaster is only powered for the lift to the top of the track, we can calculate how much electrical energy must be used to raise the car. This, along with estimates of repairs and other costs, can help the proprietor of the park determine a reasonable price to charge per ride.

figure ap.12 Entry to an amusement park.

The energy required to raise the car to the top of the ride must be equal to the gravitational potential energy, and kinetic energy if any, gained by the car as it is being raised. This gravitational potential energy is $E_p = mgh$, as discussed previously. This energy is supplied by an electric motor. The energy required by the motor will be equal to the gain in potential energy, plus a little kinetic energy to start the car rolling, plus the energy lost through friction, plus the energy losses in the motor itself. No motor is 100% efficient at converting electrical energy to kinetic energy. They all lose energy to the environment through heat, sound, and wear. A good electrical motor will only be about 20–30% efficient, and many are not as good as this. Therefore:

$$(E_{\text{into motor}}) \times 20\% = E_{\text{p gained}} + E_{\text{k gained}} + E_{\text{lost through friction}}$$

$$E_{\text{into motor}} = (mgh + E_{\text{k gained}} + E_{\text{lost through friction}}) \times \frac{100}{20}$$

The energy lost through friction has been mentioned previously as being too difficult to calculate, but we can estimate that it will be some fairly constant proportion of the potential energy gained. Higher gains will have higher losses. We could go to an existing roller coaster and measure the frictional losses for calculations in our own design. For the sake of these calculations, let us assume that 15% of the energy taken to raise the car to the top of the track will be lost through friction.

$$E_{\text{into motor}} = [mgh + E_{\text{k gained}} + 15\%(mgh)] \times \frac{100}{20}$$

Let us also assume that only a very small kinetic energy gain is necessary to start the car rolling down the track, and ignore that also. The equation then simplifies to:

$$E_{\text{into motor}} = 5.75 \times mgh$$

Since quicker change in energy requires more power, we can pick a reasonable time for the car to be towed up to the top of the ride. (Note that the longer this takes, the less power will be required.) We could then estimate the power requirements of the motor, its minimum size, and what the power bill is likely to be. Electricity costs about 15 cents per kilowatt hour. Calculating the power bill is done by multiplying the cost of electricity times the amount of electricity (electrical energy) used:

$$P = \frac{E}{t} \Rightarrow \text{Cost} = \text{Electricity rate} \times P \times t$$

* See Power—mechanical on page 516.

In order to turn in the circle, a force must act to pull them into the centre. On a carousal, this is a sequence of forces that is involved with the child holding onto the horse, the horse being fastened to the turning platform, and the platform being strong enough to take the strain.

If we assume that the weakest link in that sequence is the child holding on, we can calculate a maximum safe speed for the carousel. There are two approaches: we could estimate the maximum force that a small child can comfortably use to hold on for a long period of time, and use this to calculate the circular velocity given the radius of the ride. Alternatively, we could estimate the maximum desirable impact force should a child fall off and use this to calculate a maximum velocity. Ideally, we should use both approaches and take the lowest value as the safe speed.

Since a carousel turns in a circle, the centripetal force equation applies:

$$F_c = \frac{mv^2}{r}$$

When calculating the maximum safe velocity, this equation rearranges to give:

$$v_{max} = \sqrt{\frac{F_{max}r}{m}}$$

The mass, m, in this equation is the mass of the child riding the carousel. Note that the value of $\frac{F}{m}$ in this equation is also the acceleration, so the equation becomes:

$$v_{max} = \sqrt{a_{max}r}$$

The use of a maximum acceleration allows a general answer, instead of having to estimate a series of forces and masses for the children. Given that a car accelerates at less than 0.1g for normal accelerations, a reasonable maximum acceleration for our carousel horse would be less than this. In a car a child has a seat belt, but there are no seat belts on a carousel!

When calculating the force to be withstood by the attachments holding a horse onto a carousel, or restraints on any other ride, safety standards usually require that the actual force that can be withstood must be several times larger than what is predicted. This is called a *safety factor*. A safety factor of 10 means 10 times the expected force must be able to be withstood.

In estimating the maximum desirable impact force, we can assume that this is a function of impact velocity. Now, if the child were to fall off a chair, or ladder, they would accelerate under the action of gravity, so their impact velocity would be $v^2 = 2gs$ assuming that they are not falling initially, and that they are s metres off the ground, and $a = g$.

In a worst-case fall off a carousel horse, the child would travel in a straight line at a velocity equal to the turning velocity of the carousel. This would be a straight line when viewed from above, but the child will fall vertically also. There are several alternatives when impacting: the child could hit a post or something, or fall onto the ground and slide or bounce. The maximum velocity at which a child could 'hit' an obstruction after falling off the carousel would be a combination of the velocity of the carousel and the velocity gained by falling vertically from the height at which the child was riding.

When determining what would be an acceptable impact velocity, we could look at simpler situations; for example, a child falling off a chair would probably not be seriously injured, but from a roof undoubtedly would be. We can therefore calculate the velocity at which a child would be travelling when falling from a height, such as a chair, and use this to calculate the maximum safe velocity for the turning of the carousel.

Exercises

E28 The designer of a small child's carousel with a diameter of 6.0 m would like the maximum acceleration experienced by the child to be less than 0.02g. What is the maximum speed it should turn at?

E29 A very young relative wants a ride on a carousel with three columns of horses. If the main consideration is that they might not be able to hold on very well, should you recommend that the relative ride on the:

A inside column?
B middle column?
C outside column?

E30 On the same carousel as in exercise E29, an older and stronger relative wants a fast ride. What would you recommend for them?

E31 In designing the attachment of a 20 kg horse onto a carousel designed to carry riders up to 50 kg mass, and at a centripetal acceleration of 0.05***g***, what force does the attachment have to be able to withstand, if local regulations require a safety factor of 12?

E32 If a designer finds through research of accident reports at a hospital that children under 5 can usually survive a fall of 1 m without serious injury, what is the maximum safe turning speed for a carousel to be used by children of this age?

Spin dry cycle!

For older children, there might be more excitement on a carousel in being spun even faster. There are a variety of rides based on the principle of spinning very fast. Using the same equations as for a carousel, we could design a drum, in which children could stand, and be spun, much like being inside a spin drier. They would feel as though they were being thrown against the wall of the drum by inertia. (Note again, 'thrown' is not the right word.) If the spin was fast enough, the centripetal force added to the gravitational force could make it seem as though the wall of the drum were the floor.

Alternatively, chairs hung from chains could be spun. The resultant inertial effect would have the chairs being lifted off from vertical at an angle determined by the speed of spin and the radius of curvature.

In both cases, the centripetal force is the vector sum of the force exerted by the chair or wall and the gravitational force.

* See Vectors on page 563.

Let us first look at designing a ride that consists of a spinning bowl with a flat floor and sloping walls that the riders lean against. Can we design a ride that would make the walls appear to be the floor? Let the angle of the wall away from vertical be θ, the radius of the bowl be *r*, the speed of rotation be ***v***, and the mass of the rider be *m*. The force exerted by the wall is the vector sum of the centripetal force ad the force of gravity (see figure ap.15).

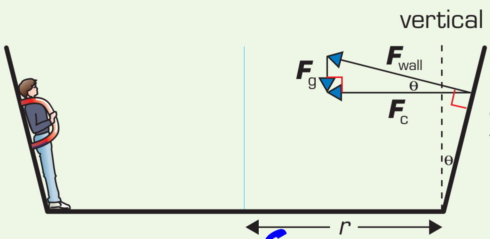


figure ap.15 The spinning bowl ride, as previously described.

Using Pythagoas's theorem, the magnitude of the force exerted bt the wall is:

$$\boldsymbol{F}^2_{\text{wall}} = \boldsymbol{F}_c^2 + \boldsymbol{F}_g^2$$

$$\boldsymbol{F}_{\text{wall}} = \sqrt{\boldsymbol{F}_c^2 + \boldsymbol{F}_g^2} = \sqrt{\left(\frac{m\boldsymbol{v}^2}{r}\right)^2 + (m\boldsymbol{g})^2}$$

This force will occur at an angle θ away from the horizontal. The value of θ is given by:

$$\tan\theta = \frac{\boldsymbol{F}_g}{\boldsymbol{F}_c} = \frac{r \times m\boldsymbol{g}}{m\boldsymbol{v}^2} = \frac{r\boldsymbol{g}}{\boldsymbol{v}^2}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{r\boldsymbol{g}}{\boldsymbol{v}^2}\right)$$

This gives us the angle at which to slope the wall away from the vertical so that the riders can walk on the wall when the ride is at full speed. During the speeding up process, the apparent 'down' will move from vertical to perpendicular to the wall, and time must be allowed for the riders to move from one position to the other, and vice versa at slowing down. Note also that the riders will feel much heavier when standing on the wall (since the resultant force is the combination of the inertial effect and gravity). This will place a limit on the safe speed of the ride (to avoid a blackout), and therefore on the maximum angle away from horizontal that the wall can be.

The calculation of the force experienced by the rider depends on the mass of the rider, so again it is easier to calculate the acceleration in ***g***-force, which is independent of mass. The acceleration experienced by the rider is the centripetal acceleration and is:

$$\boldsymbol{a} = \frac{\boldsymbol{v}^2}{r}$$

The angle for the acceleration is the same as that for the centripetal force; that is, towards the centre.

Exercises

E33 a What resultant force will be experienced by 60 kg riders in a drum of radius 5 m spinning at 4 m s^{-1}?

b In order to stand perpendicularly on the wall of the drum, what angle will the wall have to be constructed at?

E34 a If a spinning drum ride cannot exceed a 10 m radius, and the riders are not to experience an acceleration greater than 2$\boldsymbol{g}$, what is the maximum speed at which the drum can spin?

b What angle of wall in the drum will allow the riders to stand perpendicularly on it?

Bungee!

The physics of the bungee jump appears simple. The jumper starts off with a gravitational potential energy $E_p = mgh$, where m is the mass of the jumper (kg), $\boldsymbol{g}$ is the gravitational acceleration (9.8 m s^{-2}), and h is the height above a reference point (m). With a bungee jump, it is probably best to call their starting position zero, so that they acquire an increasingly negative gravitational potential energy as they drop, since their height during the jump is below the zero of their starting point. Alternatively, you could choose the lowest point of the jump as a zero reference point, but only if you know where that will be.

figure ap.16 Bungee jumping has gained popularity in recent years.

The bungee cord acts as a giant spring, absorbing energy as it stretches. There is a two-stage process. First, the jumper falls a certain distance determined by the 'slack' or loop in the bungee cord. During this phase, the jumper is effectively in free-fall, so their gravitational potential energy is transformed into kinetic energy. Second, once the slack in the cord is taken up, the jumper experiences a deceleration and the kinetic energy is transformed into elastic potential energy, stored in the stretch of the bungee cord. At the same time, there is further gravitational potential energy to kinetic energy transformation, as the jumper continues to fall further while slowing down. This means that the analysis of the physics involved can get complicated.

Fortunately, an energy analysis allows us to go directly from the gravitational potential energy to elastic potential energy. The energy transformation during the drop is:

$$E_{\text{p, gravitational}} \rightarrow E_k \rightarrow E_{\text{p, elastic}}$$

$$mgh \rightarrow \frac{1}{2}mv^2 \rightarrow \frac{1}{2}kx^2$$

and the reverse happens when the jumper bounces back up. The total energy, in theory, remains the same, but in reality there is an energy loss due not only to friction (air resistance), but also to permanent deformation of the bungee cord, so the jumper will not rise back up to the same level as the starting point. The deformation of the cord means it does not return to exactly the same length, and also the cord crystallises slightly, so it is not as stretchy as it was. This crystallisation also means that the cord is slightly more likely to break next time it is used. These two effects mean that the cord has to be replaced regularly, for safety.

The total distance that the jumper will fall, h, will depend not only on the stiffness of the bungee cord, which is the spring constant, k, but also on the length of the bungee cord at the

See Energy and work on page 379. See Potential energy on page 509.

start, l. The amount that the cord stretches is the spring extension, x. Since energy must be conserved, and if we ignore friction, then at the bottom of the jump, when the jumper momentarily comes to rest, the amount of energy stored in the spring (cord) must equal the difference between the gravitational potential energy at the start and at this point, so:

$$m\mathbf{g}(l + x) = \frac{1}{2}kx^2$$
$$m\mathbf{g}l + m\mathbf{g}x = \frac{1}{2}kx^2$$
$$\frac{1}{2}kx^2 - m\mathbf{g}x - m\mathbf{g}l = 0$$

This is a quadratic equation with respect to x, and can be solved using the formula for the solution to a quadratic equation: if $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. So:

$$\frac{1}{2}kx^2 - m\mathbf{g}x - m\mathbf{g}l = 0$$
$$\Rightarrow x = \frac{m\mathbf{g} \pm \sqrt{(m\mathbf{g})^2 + 2km\mathbf{g}l}}{k}$$

Since the expression $\sqrt{(m\mathbf{g})^2 + 2km\mathbf{g}l}$ will always be greater than $m\mathbf{g}$, there will be two solutions to the quadratic: one positive and one negative. The negative value has no meaning, and can be ignored. Therefore:

$$x = \frac{m\mathbf{g} + \sqrt{(m\mathbf{g})^2 + 2km\mathbf{g}l}}{k}$$

This equation allows us to calculate the extension of the bungee cord (given the mass of the jumper and the length and spring constant of the cord). The total distance fallen is equal to $l + x$, or:

$$\text{Total distance fallen} = \frac{m\mathbf{g} + \sqrt{(m\mathbf{g})^2 + 2km\mathbf{g}l}}{k} + l$$

Note that, in this situation, mass *does* matter, and obviously a heavier person is going to stretch the cord further. This means that before jumping, it is necessary to carefully weigh the jumper, and measure out the length of cord. A mistake in doing either may result in a shorter than possible jump (not really a problem), or the jumper hitting the ground (ouch!).

The spring constant of a bungee cord is typically in the order of 50–200 N m^{-1}, the lower values being preferable from the point of view of lower impact forces, but resulting in greater stretching and longer falls. The spring constant of the bungee cord will gradually increase with usage, as the cord crystallises and gets stiffer. An older cord should therefore supply a greater slowing force, and stop the jumper sooner rather than later. An older cord also has a greater chance of breaking! It is easy enough to test the spring constant of a cord by experimentation, in the same way as you can measure the spring constant of a spring in a laboratory.

See Hooke's law on page 407.

The maximum velocity that the jumper reaches is not easy to calculate. It will occur at an equilibrium point where the slowing force supplied by the bungee cord is equal to the gravitational force. It is easier to calculate the jumper's speed just before the bungee cord starts slowing the jumper down. This can be calculated in the same way as calculating the speed of the roller coaster in the first section of this context.

$$\mathbf{v}_{max} = \sqrt{2\mathbf{g}l}$$

In this equation, $\mathbf{v}$ is the velocity that the jumper reaches just before slowing down (m s^{-1}), $\mathbf{g}$ is the gravitational acceleration (9.8 m s^{-2}), and l is the length of the bungee cord (m).

The maximum force experienced by the jumper will occur at the maximum extension of the cord. Hooke's law states $\mathbf{F} = -kx$, which gives the maximum force when x equals the maximum extension. The negative sign indicates that the force is in a direction opposite to the extension.

See Hooke's law on page 407.

The maximum acceleration can be found from the maximum force by dividing by the mass of the person jumping, since $\mathbf{a} = \frac{\mathbf{F}}{m}$.

See Newton's second law of motion on page 486.

Exercises

E35 a An 80 kg man is going to jump off a bridge while bungee jumping. The bungee cord has a spring constant of 100 N m^{-1}, and is 20 m long. How far will he fall?
b At what speed would the bungee jumper be falling just before the cord starts to slow him down?
c What maximum force will he experience during the fall? At what point during the jump will it occur?
d What acceleration, in ***g***s, is occurring at the point of maximum force? At what velocity is he travelling at this point?
e The bridge is 150 m above a river. The person jumping wants to just touch the water in the river at the bottom of the jump. How long should the bungee cord be to allow this to occur?
f What is the maximum velocity that the person reaches during the jump in part **a**?

Experimental investigation

A Hooke's law experiment is usually done on a static system (one that is not moving). Design an experiment that will measure the change in force exerted on an object that is bouncing up and down on a bungee cord. (Hint: The force exerted on the object will be the same as the force exerted by the cord on whatever it is attached to.)

Slingshot

In a slingshot, a bungee cord is pulled down to the ground, a person is attached to it and it is released, catapulting the person up into the air. The cord is usually a V-shape, with the person attached to the middle so that the person doesn't hit the towers supporting the cord. It is easiest to deal with the mathematics of a single cord and treat it as the opposite of a bungee jump. An important difference, however, is that there is no free-fall phase of the ride, so the equations are a bit simpler.

With the cord fully stretched, there is an elastic potential energy $E_p = \frac{1}{2}kx^2$, which will be transformed into kinetic energy and gravitational potential energy. It is important to note that at the top of the ride, the cord is also stretched, so overall the conservation of energy equation gives us:

$$E_{\text{elastic, at start}} = E_{\text{elastic, at top}} + E_{\text{gravitational, at top}}$$

Let the distance that the cord is stretched down be x_d, and the distance that it is stretched up be x_u, so the overall height gain will be $(x_d + x_u)$. The energy balance equation then becomes:

$$\frac{1}{2}kx_d^2 = \frac{1}{2}kx_u^2 + m\boldsymbol{g}(x_d + x_u)$$

$$\Rightarrow \frac{1}{2}kx_d^2 - \frac{1}{2}kx_u^2 = m\boldsymbol{g}(x_d + x_u)$$

$$\Rightarrow \frac{1}{2}k(x_d^2 - x_u^2) = m\boldsymbol{g}(x_d + x_u)$$

$$\Rightarrow \frac{(x_d^2 - x_u^2)}{(x_d + x_u)} = \frac{2m\boldsymbol{g}}{k}$$

$$\Rightarrow \frac{(x_d - x_u)(x_d + x_u)}{(x_d + x_u)} = \frac{2m\boldsymbol{g}}{k}$$

$$\Rightarrow x_d - x_u = \frac{2m\boldsymbol{g}}{k}$$

$$\Rightarrow x_u = x_d - \frac{2m\boldsymbol{g}}{k}$$

Since the total height $= x_u + x_d$, then:

$$\text{Total height} = \left(x_d - \frac{2m\boldsymbol{g}}{k}\right) + x_d$$

$$= 2x_d - \frac{2m\boldsymbol{g}}{k}$$

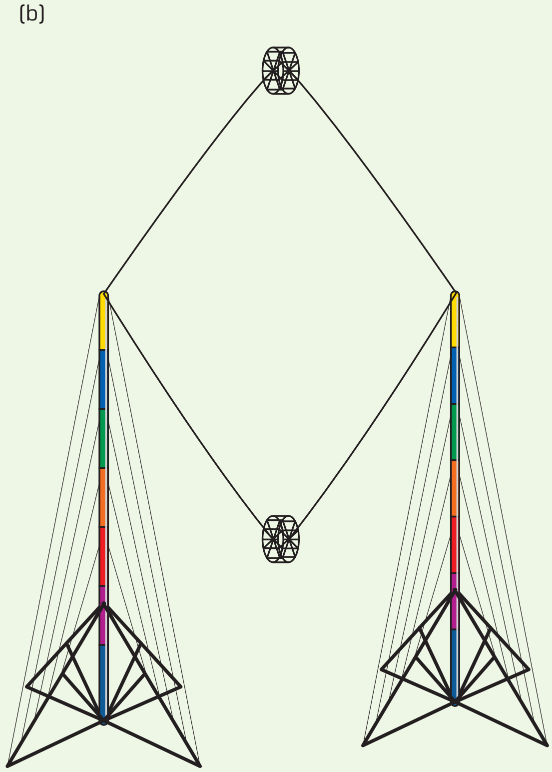


figure ap.17 The slingshot ride.

This latter equation tells us that the total distance that the person is thrown into the air will be less than twice the distance to which the elastic cord is pulled down from its rest position. This makes sense, as there will be a gain in gravitational potential energy as the person rises. In the absence of friction (air resistance), the person will fall to the start position. In reality, energy will be lost to the system through friction, so there will be insufficient energy left to fully stretch the cord back to its starting position. The overall path of the person will be a series of bounces which decrease in height, eventually coming to rest at an equilibrium position where the stretch in the cord counteracts the force of gravity. The rider can then be hauled back to the ground and released.

The limiting factor in the design of this ride is going to be the maximum force that can be experienced by the rider. The maximum force will be a combination of the force exerted by the cord at its fullest extension and the gravitational force, and will occur at the bottom or top of the ride.

$$\boldsymbol{F}_{\text{max}} = \boldsymbol{F}_{\text{spring}} \pm \boldsymbol{F}_{\text{g}} = -kx \pm mg$$

Given that there will be energy losses due to friction during the ascent, the actual maximum force will be at the start of the ride, at the bottom, just as the rider is fired into the air. Also note that at the top of the ride, the rider may feel a net force acting downwards that is greater than gravity. Due to the inertial effect discussed in detail under the roller coaster section at the start of this context, the rider will interpret this as a 'negative gravity' that may result in red-out or vomiting. At the bottom of the ride, the effect is of greatly increased weight. The time it takes to cycle from the top to the bottom is a constant, and is given by the equation of period for a vibrating mass:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where T is the period of the swing (s), m is the mass of the person and cord (kg), and k is the elastic constant of the spring (N m^{-1}). The ride will be similar to bouncing on a giant trampoline without the security of the trampoline underneath the rider, and creates large swings in the apparent acceleration suffered by the rider.

* See Pendulum on page 501.

Exercises

E36 The cord used in a slingshot ride has a spring constant of 75 N m^{-1}. The cord is stretched down from its rest position by 20 m. An 80 kg person is attached to the cord, and the cord is released. How high into the air will the rider be thrown?

E37 If a slingshot ride is made with a cord that has a spring constant of 80 N m^{-1}, and a 60 kg rider is not to experience an acceleration of greater than $2g$, how far can the cord be pulled down from its rest position in order to start the ride?

E38 A 70 kg person is riding a slingshot ride that has a spring constant of 95 N m^{-1}. The weight of the cord is negligible. Frictional forces are small. If the cord was pulled down 15 m at the start of the ride, how often will the rider reach the very top of the ride and start to fall?

E39 A slingshot ride cord has a spring constant of 100 N m^{-1}, and is pulled down 30 m from its rest position to start.

a What acceleration will be experienced by a 50 kg rider?

b Is this acceleration safe?

E40 A theme-park owner wants a really scary ride, and insists on a slingshot that will have a total height gain of 100 m. Is it possible to design such a ride and still have the rider experience no acceleration greater than $1.5g$? Justify or explain your answer.

E41 Another theme-park owner wants a slingshot for smaller children, and asks you to design one that will not result in an acceleration greater than $1g$. Is this possible, and if so, how, or if not, why not?

Spin and turn and up and down

Imagine sitting in a freely rotating bucket seat that is attached to a spinning platform that is attached to a larger spinning platform. This whirly-twirly type of ride is quite common in fairgrounds, perhaps you have even ridden one. The motion you feel when on this ride is simple to analyse. First, you have a centripetal force and acceleration created by the turning of the large platform. The large platform usually turns at a constant rate, so this aspect of the sensation is constant.

figure ap.18 Some rides are just more complicated versions of others.

The smaller platform also gives a centripetal force and acceleration. These are added to the force and acceleration provided by the larger platform, remembering that force and acceleration are both vectors, and must be added as vectors. At times, the two will reinforce each other, giving a maximum (which is the sum of the two) and at other times will oppose each other, giving a minimum (which is the difference). Either force could be the larger, so the difference may result in forces and accelerations in the opposite direction to the maximums.

The times at which the two forces oppose or reinforce each other will depend on the periods of motion of each; but assuming constant rates of rotation for each, then these will be predictable and regular.

For even more excitement, the larger platform can be made to raise and lower, or tilt. This adds a vertical component to the sensation, and if the tilt is far enough, it adds a gravitational force component to the maximum and minimums mentioned before!

The analysis of the motion of this ride can be quite complex, so we will limit ourselves to calculating the maximum force and acceleration in the exercises.

Exercises

E42 In a whirly-twirly ride, the small platform has a radius of 2 m and rotates at $0.5\ \mathrm{m\ s^{-2}}$ and the large platform has a radius of 10 m and rotates at $2\ \mathrm{m\ s^{-2}}$. What is the maximum force experienced by a 50 kg rider?

E43 In another whirly-twirly ride, the small platform has a radius of 2 m and rotates at $1\ \mathrm{m\ s^{-2}}$ and the large platform has a radius of 10 m and rotates at $0.6\ \mathrm{m\ s^{-2}}$. What is the maximum acceleration experienced by the riders?

Extended response

Design your favourite type of amusement park ride. Your design should take into consideration the maximum forces and accelerations experienced by the riders and contain a detailed account of what the riders will experience during the course of the ride.

Activity

Build a scale model of an existing theme park ride, or design a ride yourself and build a model of it to check your design. Your ride does not have to be built using model cars and people. For example, you could build a model roller coaster using clear plastic tubing and marbles.

Experimental investigations

1 Visit a theme park and take measurements to determine the force and acceleration experienced by a rider. Compare your results with your classmates to determine which ride results in the greatest force or acceleration experienced by riders.

2 Analyse an existing theme park ride in detail and calculate the forces and accelerations experienced by the riders. This might involve taking measurements of an existing ride during a visit to a theme park, or accessing the data another way. Your calculation of the forces and accelerations might be theoretical or practical. That is, you might take measurements of heights and slopes, and calculate the theoretical forces and accelerations, or you might take an accelerometer on the ride and measure the forces that way. If you are able to, do both and compare the theoretical results with the practical measurements.