

THE LEADING **EDGE**

**Ready for
Mathematical Methods 3 & 4
CAS**

Anne Matheson
Kylie Boucher

hi.com.au/theleadingedge

Contents

Introduction to Ready for book and iMaths CD	1
Using your CAS calculator for Mathematical Methods 3 & 4 CAS	2
Chapter 1 Algebra preparation	10
1.1 Substitution	10
1.2 Transposition	11
1.3 Solving linear equations	13
1.4 Algebraic fractions	15
1.5 Equations involving fractions	19
1.6 Expansion	20
1.7 Simultaneous equations	24
1.8 Absolute value	27
1.9 Index laws	29
1.10 Logarithmic laws	32
1.11 Factorisation of quadratics	35
1.12 Completing the square	38
1.13 Factorising using completing the square	41
1.14 Difference of two squares	43
1.15 Sum and difference of perfect cubes	44
1.16 Solving equations using the null factor law	45
1.17 Remainder and factor theorem	48
1.18 Finding angles and sides using trigonometry	50
1.19 Exact value triangles	52
1.20 Solving trigonometric equations	53
1.21 Distance between two points	56
Chapter 2 Functions and graphs preparation	57
2.1 Function notation	57
2.2 Domain and range	58
2.3 Interval notation	59
2.4 Linear graphs	61
2.5 Finding the gradient of a straight line	62
2.6 Finding the equation of a straight line	64
2.7 Equations of parallel and perpendicular lines	66
2.8 Sketching quadratic graphs	69
2.9 Maximum and minimum turning points	73
2.10 Sine and cosine graphs	74
2.11 Tangent graphs	75
2.12 Standard graphs	77
2.13 Finding the equation from the graph	84
2.14 Inequalities	87

Chapter 3 Calculus preparation	89
3.1 Average rate of change	89
3.2 Basic differentiation	90
3.3 Instantaneous rate of change	92
3.4 Equation of a tangent	93
Chapter 4 Probability preparation	94
4.1 Continuous and discrete data	94
4.2 Finding probabilities	95
4.3 Independent events	97
4.4 Arrangements	100
4.5 Tree diagrams	102
4.6 Addition rule	108
4.7 Conditional probability	110
Notes	112
Tear-out Answers	117

Introduction

Are you going to study Mathematical Methods 3 & 4 CAS next year?
Have you just started Mathematical Methods 3 & 4 CAS?
Are you doing Mathematical Methods 3 & 4 CAS but struggling with it?

If you answered yes to any of these questions then *The Leading Edge: Ready for Mathematical Methods 3 & 4 CAS* and iMaths CD are for you.

It is no use trying to get to grips with Mathematical Methods 3 & 4 CAS if you haven't got the building blocks you need. If there is something you didn't quite understand in previous years (or if you need to practise a fundamental skill a little more) then you'll find it here. All the skills and concepts you need to be prepared for this subject are covered in this book.

The Mathematical Methods 3 & 4 CAS course is divided into areas of study. The four chapters of this book cover the preparation for each of the areas of study. You need to answer questions on all the areas of study during the end-of-year examination, so it would be a good idea to work through all the material provided in the book.

At the front of this book there is a section that provides clear instructions on the use of the TI and Casio CAS calculators to complete the functions required in the Mathematical Methods 3 & 4 course. Each of the chapter sections of *Ready for Mathematical Methods 3 & 4* has clear information and examples, followed by questions of increasing difficulty and space to write the step-by-step solutions. At the end of the book are the tear-out answers to all the questions.

The accompanying iMaths CD provides you with eTutorials that offer animated, narrated presentations of the key concepts and skills you will need.



Note that *Ready for Mathematical Methods 3 & 4 CAS* and the accompanying iMaths CD do not cover the Mathematical Methods 3 & 4 CAS course content. They help to ensure you are ready for the course.

For help with Mathematical Methods 3 & 4 CAS course content, see the following titles in the Leading Edge series:

- *The Leading Edge: Mathematical Methods 3 & 4 CAS Pocket Study Guide*
- *The Leading Edge: Mathematical Methods 3 & 4 CAS Exam 1 Builder*
- *The Leading Edge: Mathematical Methods 3 & 4 CAS Exam 2 Builder*

Visit us at hi.com.au/theleadingedge

1.11 Factorisation of quadratics

Worked example 1

Factorise $x^2 - 11x + 28$.

Steps

1. Determine the factors of x^2 and +28 which when added together give -11x.
2. Put into brackets to give the solution.

Solution

Factors of x^2 are x and x . Factors of +28 that when added give -11 are -4 and -7.

$$\begin{aligned} x^2 - 11x + 28 \\ = (x - 7)(x - 4) \end{aligned}$$

Worked example 2

Factorise $-2x + 8x^2 - 15$.

Steps

1. Rearrange the expression to put the highest power first.
2. Since $8 \times -15 = -120$, determine factors of -120 that when added together give -2.
3. Write the expression using brackets and dividing by 8 (the coefficient of x^2).
4. Factorise and simplify.

Solution

$$8x^2 - 2x - 15$$

Factors of -120 are:
40, -3; 12, -10; -12, 10; etc.
The combination that multiply to give -120 and add to give -2 is -12, 10.

$$\begin{aligned} \frac{(8x + 10)(8x - 12)}{8} \\ = \frac{2(4x + 5) \times 4(2x - 3)}{8} \\ = (4x + 5)(2x - 3) \end{aligned}$$

Worked example 3

Factorise $-12 - 27x^2 + 36x$.

Steps

1. Rearrange the expression to put the highest power first.
2. Take out common factor of -3.
3. Determine factors of $9x^2$ and 4 which when added together give -12x.
4. Put into brackets to give the solution.
5. As the term $3x - 2$ is repeated, solution is

Solution

$$-27x^2 + 36x - 12$$

$$-3(9x^2 - 12x + 4)$$

Factors of $9x^2$ are $9x$, x and $3x$, $3x$. Factors of 4 are 1, 4; 2, 2; -2, -2 and -1, -4. The combination which gives -12x is

$$\begin{aligned} 3x \times -2 &= -6x \text{ and } 3x \times -2 = -6x. \\ -6x + -6x &= -12x \end{aligned}$$

$$\begin{aligned} -27x^2 + 36x - 12 \\ = -3(3x - 2)(3x - 2) \\ = -3(3x - 2)^2 \end{aligned}$$

Example 4

Factorise $12 - 4x - x^2$.

Solution

Factors of 12 are 12×1 , 6×2 and 4×3 . The factors of $-x^2$ are $-x \times x$.

Using trial and error, combine these terms so that they add to $-4x$.

$$6 \times -x = -6x \text{ and } 2 \times x = 2x$$

$$-6x + 2x = -4x \therefore 12 - 4x - x^2 = (6 + x)(2 - x)$$

Alternatively, take out the -1 as a common factor and then factorise.

$$-(x^2 + 4x - 12) = -(x + 6)(x - 2)$$

Exercise 1.11

1 Factorise the following quadratic expressions.

(a) $x^2 - 5x - 36$

(b) $x^2 - 13x + 42$

(c) $x^2 + 14x + 49$

(d) $x^2 - 20x + 100$

(e) $10x^2 + x - 3$

(f) $4x + 12x^2 - 5$

(g) $12 + 12x + 3x^2$

(h) $100 - 80x + 16x^2$

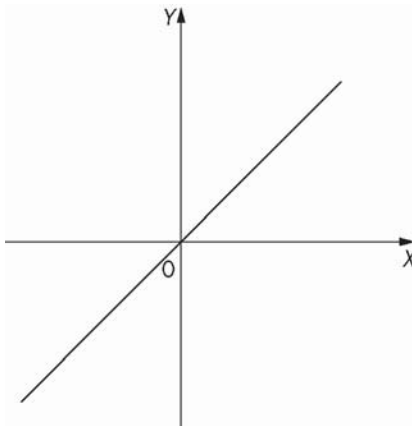
(g) $y = -\frac{1}{x+2} + 2$

(h) $y = \frac{1}{x-2} - 3$

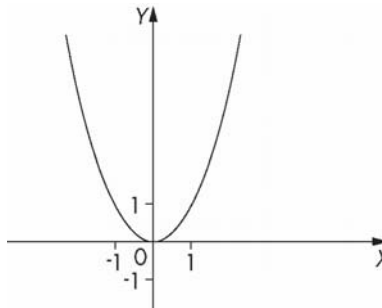
2.13 Finding the equation from the graph

To find the equation for a graph it is a good idea to already be aware of the basic shapes of different graphs of functions. Then, any transformations can be included in the equation. Some basic graphs are shown below.

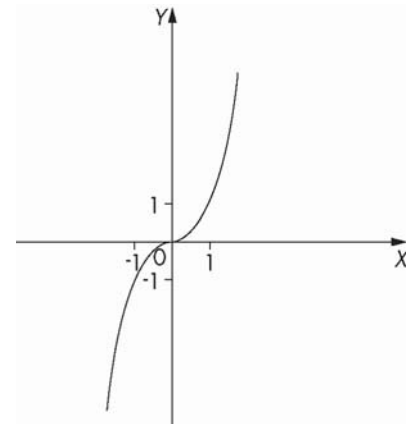
$y = x$



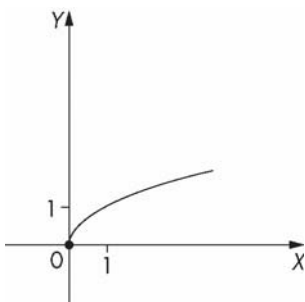
$y = x^2$



$y = x^3$

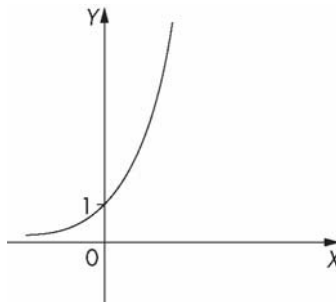


$y = \sqrt{x}$



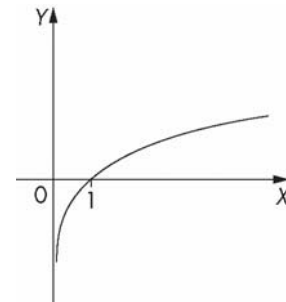
$y = 10^x$

asymptote at $y = 0$

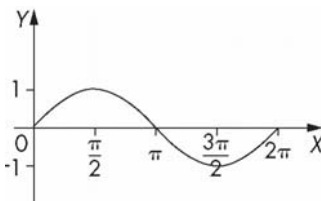


$y = \log_{10} x$

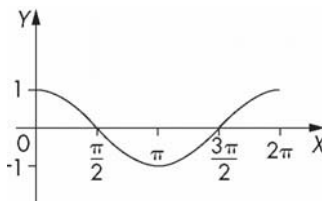
asymptote at $x = 0$



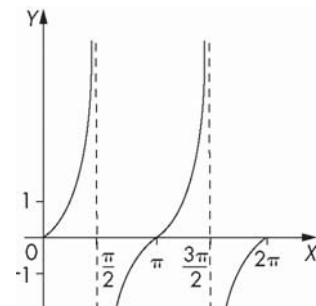
$y = \sin x$



$y = \cos x$

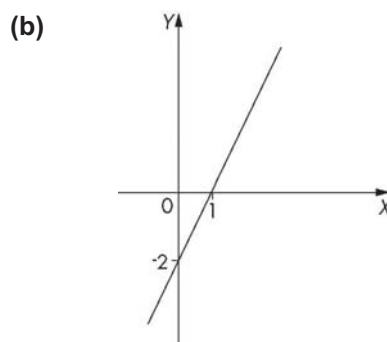
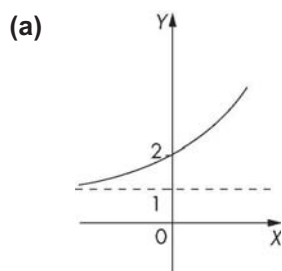


$y = \tan x$



Worked example

Find the equation for each of the following graphs.



Steps

- (a) 1. Identify the basic graph shape.
 2. Note any transformations.
 3. Write the required equation.

- (b) 1. Identify the basic graph shape.
 2. Note any transformations.
 3. Write the required equation.

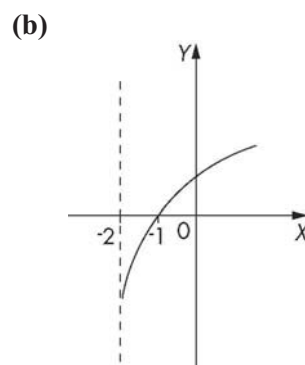
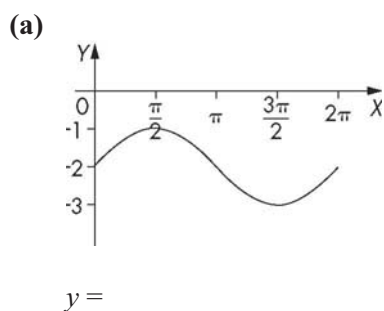
Solutions

- (a) Basic graph shape is $y = 10^x$
 translated up 1 unit
 $y = 10^x + 1$

- (b) Basic graph shape is $y = x$
 dilated by a factor of 2 parallel to the y-axis and
 translated 2 units down
 $y = 2x - 2$

Exercise 2.13

1 Find the equation for each of the following graphs.



4.7 Conditional probability

The probability of A given B is $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

Worked example

A normal fair die is thrown. What is the probability that, given an even number turned up, it was a multiple of 3?

Steps

1. Define the events.
2. Write out the sample space.
3. Calculate the probabilities.
4. Substitute the values into the rule

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Solution

A = getting a multiple of 3
 B = getting an even number

A = {3, 6}
 B = {2, 4, 6}
 A ∩ B = {6}

$$\Pr(A) = \frac{2}{6} = \frac{1}{3}$$

$$\Pr(B) = \frac{3}{6} = \frac{1}{2}$$

$$\Pr(A \cap B) = \frac{1}{6}$$

$$\Pr(A|B) = \frac{1}{6} \div \frac{1}{2} = \frac{1}{3}$$

Exercise 4.7

- 1 (a) A fair die is thrown and an even number appears uppermost. What is the probability that it is a 4?
- (b) A coin is tossed three times and it is noticed that the coin lands on tails exactly twice. What is the probability that it is the third toss which produces the head?
- (c) A pair of fair dice are rolled. Find the probability that both numbers are even given that the second die shows a number greater than 3.
- (d) Two coins are tossed. What is the probability that both coins land as tails, given that at least one coin lands as tails?