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General Further Mathematics

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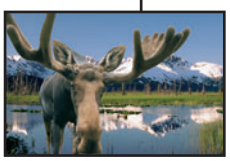

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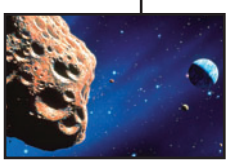

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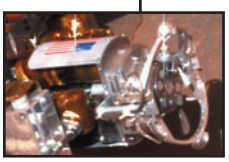

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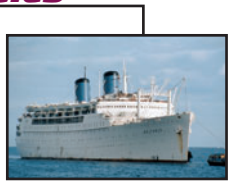

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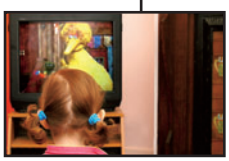

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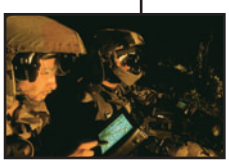

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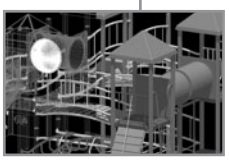

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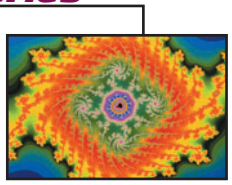

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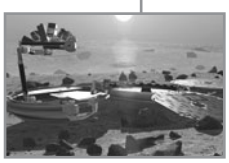

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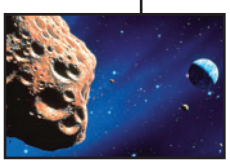

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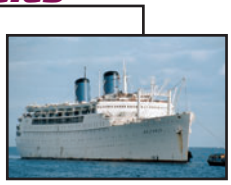

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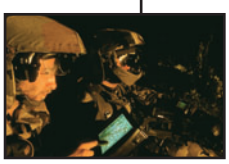

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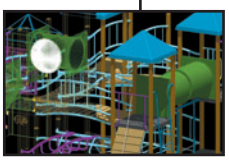

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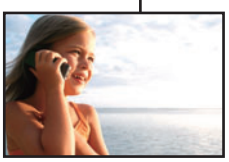

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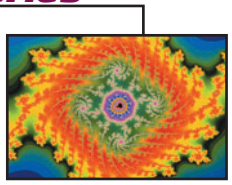

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prepzone 12

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, click on the Replay Worksheet icon on your Student CD or ask your teacher for the Replay Worksheet. Fully worked solutions to *every* question in this Prep Zone are contained in the Student Worked Solutions book. See the order form at the back of this textbook or go to www.hi.com.au/vcezonemaths for further details.

Worksheet R12.1

1 Solve for x in the following equations.

(a) $x - 5 = 12$ **(b)** $4 + 3x = 19$ **(c)** $2 + 4(x - 1) = 26$

Worksheet R12.2

2 Substitute $n = 6$ into each of the following and evaluate t.

(a) $t = 23 - 2n$ **(b)** $t = 2(n - 6)^2$ **(c)** $t = 25 + 3(n + 2)$

Worksheet R12.3

3 Solve each of the following pairs of simultaneous equations.

(a) $a + 3d = 15$
$a + 5d = 29$

(b) $a + 5d = 3$
$a + 14d = -24$

Worksheet R12.4

4 Simplify each of the following.

(a) $\frac{1}{5} - \frac{1}{3}$ **(b)** $3\frac{2}{3} - 2\frac{7}{12}$ **(c)** $\frac{9}{4} \div \frac{-3}{2}$ **(d)** $6 \div 5\frac{1}{4}$

Worksheet R12.5

5 Simplify each of the following.

(a) $7x + y - (6x + 2y)$ **(b)** $8p + 7q - (4p - 3q)$ **(c)** $n(2 + 4n) - n(3 - n)$

Worksheet R12.6

6 Use the quadratic formula to solve the following quadratic equations.

(a) $n^2 - 2n - 28 = 0$ **(b)** $2n^2 + n - 18 = 0$ **(c)** $484 = 4n + 6n^2$

Worksheet R12.7

7 Write each of the following to six decimal places.

(a) $2.\dot{3}$ **(b)** $0.\overline{46}$ **(c)** $5.2\overline{93}$ **(d)** $7.\overline{312}$

Worksheet R12.8

8 Simplify each of the following.

(a) $0.5 \div 0.05$

(b) $0.0043 \div 0.000\,043$

(c) $1000(1.1)^2$

(d) $\dfrac{200(1.2)^4}{1.2 - 1}$

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REPLAY FILE

To solve equations, use inverse operations on both sides of the equals sign.

Simultaneous equations can be solved by:

- substitution
- elimination.

To add or subtract fractions, find the lowest common denominator (LCD).

To multiply fractions, cancel where possible and then multiply numerators and denominators.

To divide fractions, multiply by the reciprocal.

Quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $ax^2 + bx + c = 0$.

To factorise the quadratic trinomial $ax^2 + bx + c$ look for two numbers that:

- multiply to give ac and
- add to give b.
- Once these two numbers are found, continue by using other methods of factorising, such as grouping 'two and two' and taking out common factors.

12.1 Arithmetic sequences

The finals of the Soccer World Cup are held every 4 years. This is an example of an arithmetic sequence because each pair of consecutive terms, or years, is separated by a constant value, in this case, 4 years.

This can be written in general terms where a is the 1st term, d is the **common difference** and t_1 (t one) is the name for the first term, t_2 the 2nd term, and so on. The **general term** of the sequence is called t_n.

1st term $t_1 = 1998 = a$
2nd term $t_2 = 2002 = 1998 + 4 = a + d$
3rd term $t_3 = 2006 = 1998 + 2 \times 4 = a + 2d$
4th term $t_4 = 2010 = 1998 + 3 \times 4 = a + 3d$
5th term $t_5 = 2014 = 1998 + 4 \times 4 = a + 4d$

You will notice that the number of lots of d is always one less than the term number, so if we have n terms there will be $(n - 1)$ lots of d.

From this pattern, the general term, t_n, for an arithmetic sequence with the 1st term, a, and common difference, d, is $t_n = a + (n - 1)d$.

The general term is sometimes known as the **explicit function** of the sequence.

worked example 1

Determine if the following sequences are arithmetic.

(a) 6, 10, 14, 18, …

(b) $7\frac{1}{2}, 3\frac{1}{2}, -1\frac{1}{2}, -5\frac{1}{2}$

Steps

(a) 1. Find the difference between the first two terms.

2. Find the difference between the other pairs of terms.

3. Determine if the difference, d, is the same; that is, do we have a *common* difference?

(b) 1. Find the difference between the first two terms.

2. Find the difference between the next two terms.

3. Determine if the difference, d, is the same. *Note:* We need to find only one pair where the difference is different for it not to be an arithmetic sequence.

Solutions

(a) 6, 10, 14, 18, …

$d = t_2 - t_1$
$= 10 - 6 = 4$

$d = t_3 - t_2$
$= 14 - 10 = 4$

$d = t_4 - t_3$
$= 18 - 14 = 4$

Each pair of values has the same common difference.
$\therefore$ d-values are the same.
$\therefore$ 6, 10, 14, 18, … is an arithmetic sequence.

(b) $7\frac{1}{2}, 3\frac{1}{2}, -1\frac{1}{2}, -5\frac{1}{2}$

$d = t_2 - t_1$
$= 3\frac{1}{2} - 7\frac{1}{2} = -4$

$d = t_3 - t_2$
$= -1\frac{1}{2} - 3\frac{1}{2} = -5$

d-values are not the same.
$\therefore$ $7\frac{1}{2}, 3\frac{1}{2}, -1\frac{1}{2}, -5\frac{1}{2}$ is *not* an arithmetic sequence.

dangerzone

All consecutive pairs of terms must have the same common difference in order to be considered an arithmetic sequence.

worked example 2

For the arithmetic sequence -4, 4, 12, … find t_8.

Steps	Solution
1. Determine a, the 1st term, and d, the common difference.	-4, 4, 12, … $a = -4$ $d = t_2 - t_1 = 4 - (-4) = 8$ Check: $d = t_3 - t_2 = 12 - 4 = 8$ $\therefore d = 8$
2. Substitute a and d into the formula $t_n = a + (n - 1)d$ to find the 8th term, where $n = 8$.	$t_n = a + (n - 1)d$ $t_8 = -4 + (8 - 1)8$ $= -4 + 7 \times 8 = 52$

worked example 3

Find the number of terms for the arithmetic sequence -4, 4, 12, … 156.

Steps	Solution
1. Use the values for a and d determined in Worked Example 2 and let the final term in the sequence be the nth term; that is, $t_n = 156$ and substitute into the formula.	$t_n = a + (n - 1)d$ $156 = -4 + (n - 1)8$
2. Solve for n.	$160 = 8(n - 1)$ $20 = n - 1$ $n = 21$ 156 is the 21st term.

worked example 4

In an arithmetic sequence, the 3rd term is 10 and the 20th term is -41. Determine the 1st term and the general term of the sequence. Use this general term to find the 10th term.

Steps	Solution
1. Write the formula for the 3rd term and substitute for t and n. Label this equation (1).	$t_n = a + (n - 1)d$ $t_3 = 10$ $= a + (3 - 1)d$ $10 = a + 2d$ -----(1)
2. Write the formula for the 20th term and substitute for t and n. Label this equation (2).	$t_{20} = -41$ $= a + (20 - 1)d$ $-41 = a + 19d$ -----(2)

chapter 13 REVIEW

See the Study Guide section for this chapter at the end of this textbook for a Chapter Summary (p. 646), Frequently Asked Questions (p. 647), and Study Notes (p. 648). See the Student CD for the Cumulative Practice Examinations.

Use the following to check your progress. If you need more help with any questions, turn back to the section given in the side column, look carefully at the explanation of the skill and the worked examples, and try a few similar questions from the Exercise provided. Fully worked solutions to *every* question in this Chapter Review are contained in the Student Worked Solutions book. See the order form at the back of this textbook or **www.hi.com.au/vcezonemaths** for further details.

Short answer

1 **(a)** The scale on a map has 1 cm representing 750 m. Write this scale in ratio form. **13.1**

(b) Emma's house is 2.4 km from the local pool. What length will represent this distance on the map?

(c) Reece finds the shortest route to work on the map is 62 mm. What is the actual length of this route?

2 The diameters of two circles are in the ratio 2 : 7. The diameter of the smaller circle is 4 cm. **13.1 13.2**

(a) The radii of the two circles are in what ratio?

(b) What is the diameter of the larger circle?

(c) What is the ratio of the areas?

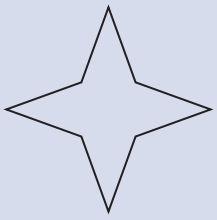
(d) Area of a circle = πr^2. Use this formula to show that your answer to part **(c)** is correct.

3 For each of the two-dimensional shapes below list: **13.3**

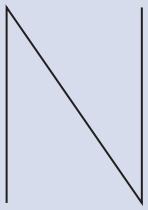
(i) the number of lines of symmetry (if any), and

(ii) the order of rotational symmetry.

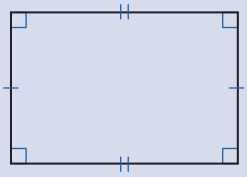
(a)



(b)



(c)



4 On eight vertices a complete graph is drawn. How many edges does it have? **13.4**

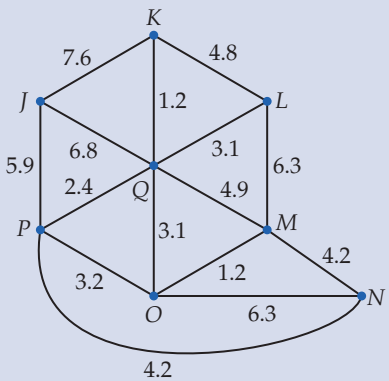
5 A planar graph with 12 edges is drawn on eight vertices. How many regions is the plane divided into? **13.5**

6 Explain why the complete graph on 19 vertices has an Euler circuit, but the complete graph on 20 vertices does not. **13.6**

7 The vertices in the network on the right represent towns and the weights represent the distances between them in kilometres. Some towns cannot be reached without going through other towns. **13.6**

(a) Find the shortest distance between town *N* and town *J*.

(b) A delivery person needs to visit every town and does not need to end at the starting town. Find a suitable path.



8 A company wants to lay cable for telecommunications between the towns in the network above. What is the minimum amount of cable required? Show your working. 13.7

Multiple choice

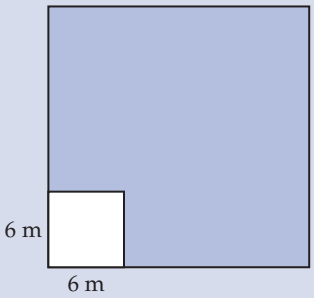
9 The scale on a map is 1 : 30 000. If the distance between two landmarks is 2.4 cm on the map, the actual distance between them is: 13.1

A 7.2 km **B** 72 m **C** 0.8 km **D** 0.72 km **E** 80 m

10 A house plan has a scale of 1 : 60. On the plan the kitchen has dimensions 4.5 cm by 5 cm. The actual area of the kitchen is: 13.2

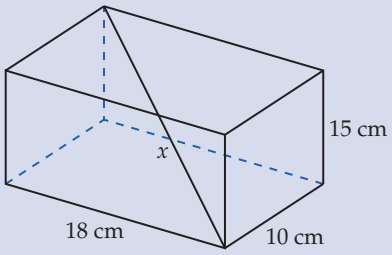
A 6.25 m^2 **B** 8.1 m^2 **C** 22.5 m^2 **D** 62.5 m^2 **E** 81 m^2

11 A square block of land has a small square vegetable patch in one corner. The shaded area represents the remaining land. The area of the land in total is 196 m^2, and the vegetable patch has a length of 6 m. The ratio of the area of the vegetable patch to the remaining land is: 13.2



A 3 : 7 **B** 6 : 40 **C** 6 : 49
D 9 : 40 **E** 9 : 49

12 The order of rotational symmetry about the axis x in this cuboid is: 13.3



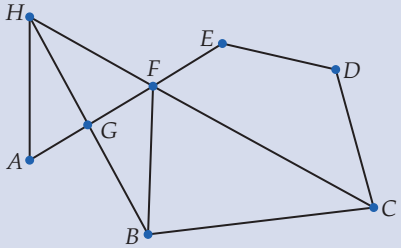
A 0 **B** 1 **C** 2
D 4 **E** 8

13 For the matrix below, which of the statements is incorrect? 13.4

	A	B	C	D	E
A	0	0	1	1	2
B	0	0	0	0	0
C	1	0	0	1	0
D	1	0	1	2	0
E	2	0	0	0	0

A There exists one isolated vertex.
B There exists one loop.
C There are no multiple edges.
D There are five vertices.
E There are six edges.

14 The sum of the degrees of the vertices in this graph is: 13.4



A 21 **B** 22 **C** 23
D 24 **E** 25

15 A planar graph with 16 edges divides the plane into seven regions. The number of vertices in this graph is: 13.5

A 7 **B** 9 **C** 11 **D** 21 **E** 23